УДК 515.2

ALGORITHM FOR SYMMETRIZATION OF A SURFACE DEFINED BY A DISCRETE POINT SKELETON

https://doi.org/10.5281/zenodo.20045776

M.M. Abduraimov
PhD, Associate Professor, SamGASU
S.A. Makhmudova
Department of Architecture, SamGASU

Annotation

The paper considers an algorithm for the symmetrization of a surface defined by a discrete point frame. The method is based on the sequential transformation of the surface sections using the Steiner symmetrization principle. An approach to redistributing discrete points using piecewise interpolation is proposed, which allows for the construction of symmetric sections relative to the specified axes. The process of forming a curved axis of symmetry and its subsequent transformation into a straight axis is described, relative to which the points are reflected. The article discusses the features of processing discontinuous sections and the possibility of multiple application of the algorithm. It is shown that when the symmetrization procedures are performed iteratively, the surface tends to approach a surface of revolution. The algorithm can be used in solving design problems where it is required to preserve volumetric characteristics and vary the geometric shape.

Keywords

Surface symmetrization, discrete point frame, Steiner symmetrization, interpolation, geometric modeling, discrete sections, axis of symmetry, rotation surface, transformation algorithms, surface design.

Introduction

In architectural and engineering design tasks, considerable attention is paid to the shaping of surfaces with specified geometric and physical characteristics. In particular, surfaces with symmetry properties play an important role, as such shapes provide not only aesthetic expressiveness, but also rational distribution of loads, as well as manufacturability of construction.

One of the effective methods for obtaining symmetric forms is symmetrization, which is based on transformations of geometric objects while preserving their integral characteristics. A classic example of this approach is

AMERICAN JOURNAL OF MULTIDISCIPLINARY BULLETIN
ISSN: 2996-511X (online) | ResearchBib (IF) = 9.512 IMPACT FACTOR
Volume-4| Issue-5| 2025 Published: |30-05-2026|

Steiner symmetrization, which is widely used in geometry and variational problems. However, its direct application to complex spatial surfaces defined by discrete data is challenging.

In modern computer-aided design systems, surfaces are often represented as discrete point meshes obtained through numerical simulations or experimental measurements. This requires the development of special algorithms that allow geometric transformations to be performed directly on discrete structures without first converting them to an analytical representation.

In this regard, the task of developing an algorithm for symmetrizing a surface defined by a discrete set of points, while preserving its geometric integrity and allowing for repeated application of transformations, is relevant.

The purpose of this work is to develop and describe an algorithm for symmetrizing a surface defined by a discrete point mesh, as well as to study its properties and applications in architectural and engineering design tasks.

The main part

Steiner symmetrization transforms shapes into symmetrical shapes while preserving the areas of flat shapes and the volumes of spatial bodies.

In those design tasks where the volume bounded by the surface plays a significant role, the variety of surface shapes (design options) can be achieved using the symmetrization transformation. The algorithm proposed below is implemented for a surface F that has a flat contour L and is defined by a discrete set of lines qj, each of which belongs to the plane Δj and is represented by a discrete frame of points Ai.j (Fig. 1).

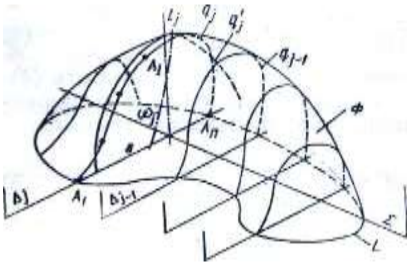

Fig. 1.

Surface symmetrization is carried out in several stages, the number of which depends on the initial data and the desired result.

The presence of a flat contour L on the surface allows us to select two points (A1 and An) belonging to L in each section qj from the set of points Ai.j. The line s passing through the points A1 and An defines the direction of symmetrization, and the line h parallel to s and touching qj divides the line qj (the set Ai.j) into two subsets that are transformed into subsets that are symmetrical with respect to the straight axis.

In cases where the discrete set of points Ai.j representing the section is not incident to a discrete set of lines parallel to the selected direction of symmetrization, it is necessary to replace one discrete set of points with another.

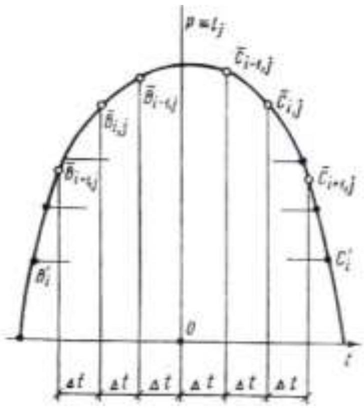

Fig. 2.

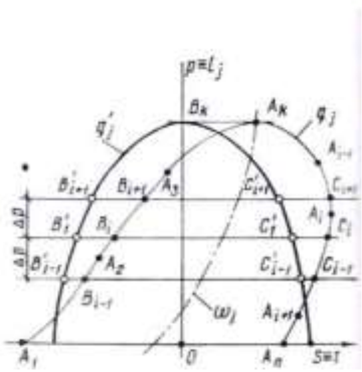

Fig. 3.

To do this, we need to describe a piecewise set of points Ai on the line qj using curves of the type

$$t = f_i(p),$$

for example, by parabolas of the third order. In this case, on each interval [Ai-1, Ai+1] (Fig.2), instead of a series of points Ai-1, Ai, Ai+1, we will obtain a new series with the required step Δp along the p-axis. The points located to the left of Ak are denoted by the letter Bi, and those located to the right of it are denoted by Ci, so that the pairs Bi-1 — Ci-1, Bi — Ci, and Bi+1 — Ci+1 on qj will be located at the same height p.

By defining the position of the curvilinear axis of symmetry ώj as a set of points that bisect the segments BiC, BiC+1, etc., and transforming the axis ώj into a straight line perpendicular to the direction of symmetrization s (in this example, the straight line lj), we obtain a transformed line qj that is symmetrical with respect to lj.

To transform the surface F into F', which is symmetric with respect to the plane Σ, which is perpendicular to Δj, it is sufficient to transform each line qj into qj, which is symmetric with respect to the line lj obtained by intersecting the plane Δj with the plane Σ (see Figure 1).

The next step, the symmetrization of the surface F' in F", can be achieved by transforming the sections of a different direction, such as those belonging to planes parallel to Σ. The series Bi and Ci on qj were previously obtained with an interval of Δp. However, in the second step, the qj line requires points with a specific interval of Δt. Therefore, it is necessary to re-describe the qj line using piecewise functions.

$p=f_i(t)$,

by piecewise interpolating the points B i and C i , and then on each line q j , determine the points B j and C j of its intersection with the planes Σ i and the planes specified with the interval Δt (Figures 3 and 4).

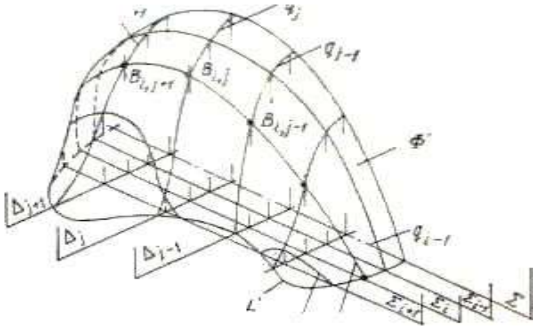

Fig. 4.

Thus, the surface F' is defined by a point array B i,j, where the index i indicates the number of the section parallel to the plane Σ, and the index j indicates the number of the section parallel to the plane Δ.

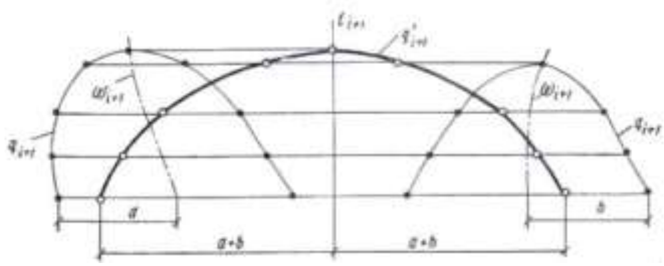

Fig. 5.

The lines qi incident to the planes Σi are symmetrized in a direction parallel to Σi and the plane of the contour L.

When symmetrizing the line qi, the procedures described for the line qi ∩ Δj are repeated. One of the planes parallel to Δj can be used as the symmetry plane.

If a line is obtained in the section of the surface by a plane (for example, Σi+1, Fig. 5), represented by two disjoint sections, its symmetrization by the proposed algorithm is possible and is carried out according to the same rule, namely: the distances (a and b) from the points on the sections of the given lines to the symmetry axes of these sections are added together; the resulting distance (a + b) is plotted from the axis (in this case, the line li+1), relative to which the given line is symmetrized.

The described symmetrization procedures can be repeated multiple times, and the current symmetry plane can be any plane perpendicular to the base plane of the surface.

If you repeat the transformations multiple times, the surface will tend to become a surface of revolution.

Conclusions

1. An algorithm for symmetrizing a surface defined by a discrete point frame has been developed, based on the sequential processing of sections and the application of the Steiner symmetrization principle.
2. It is shown that the use of piecewise interpolation allows for the correct redistribution of discrete frame points and the formation of pairs of points for subsequent symmetric transformation.
3. It has been established that the introduction of a curved axis of symmetry, followed by its transformation into a straight axis, ensures the stability and universality of the algorithm.
4. The proposed method is applicable to cases where the surface sections have a complex structure, including discontinuous sections.
5. Repeated application of the symmetrization procedure in different directions leads to a gradual smoothing of the shape and the surface tends to become a surface of revolution.
6. The algorithm can be effectively used in architectural and engineering design tasks where it is necessary to control the shape of a surface while preserving its volume characteristics.

LITERATURE:

1. Абдураимов М.М. и другие. Оптимизация безмоментно стержневых цилиндрических сводов с учетом стрелы подъема. Материалы международной научно-технической конференции ″Перспективы применения инновационных технологий в сфере архитектуры и строительства″. Самарканд,-2016 г.с.51-54.
2. Абдураимов М.М. Оптимизация геометрических параметров металлической фермы. « ИЛГОР МУХАНДИСЛАР АРХИТЕКТОРЛАР»: баркарор ривожланиш учун илм-фан ва таълимни модернизация килиш. Халкаро илмий-амалий конференсия туплами. Самарканд, 2024 й.- б.51-54.
3. Абдураимов М.М. и другие. Оптимизация безмоментно стержневых цилиндрических сводов с учетом стрелы подъема. Материалы международной научно-технической конференции ″Перспективы применения инновационных технологий в сфере архитектуры и строительства″. Самарканд,-2016 г.с.51-54.

4. Абдураимов М.М. Минимизация стоимости висячей стержневой структуры. Сборник материалов международной научно-технической конференции на тему «Вопросы устойчивого развития архитектуры и городского строительства в приаральском регионе». Нукус, 2024 г .с.175.

5. Abduraimov M.M. Some Ways of Increasing the Durability and Rigidity of Spatial Systems Based on Bionic Principles. In the book: Applied Geometry and Engineering Graphics. Kiev: KDMTUBA, 1994, issue 56, pp. I2I-I22.

6. Abdusattorovna, M. S. (2024). RELATIONSHIP OF PEDAGOGY WITH OTHER SCIENCES AND SYSTEM OF PEDAGOGICAL SCIENCES. Innovative: International Multidisciplinary Journal of Applied Technology (2995-486X), 396-398.

7. Abdusattorovna, M. S. (2023). Project Proposals for Greening and Reconstructing Modern City Streets Development. JOURNAL OF ENGINEERING, MECHANICS AND MODERN ARCHITECTURE, 2(4), 27-31.

8. Abdusattorovna, M. S. (2024). RELATIONSHIP OF PEDAGOGY WITH OTHER SCIENCES AND SYSTEM OF PEDAGOGICAL SCIENCES. Innovative: International Multidisciplinary Journal of Applied Technology (2995-486X), 396-398.

9. Abdusattorovna, M. S. (2023). Historical roots of urban gardening of streets and squares. CENTRAL ASIAN JOURNAL OF ARTS AND DESIGN, 4(2), 19-22.

10. Abdusattorovna, M. S. (2023). ZAMONAVIY SHAHARSOZLIKDA KO 'CHA VA XIYOBONLAR LANDSHAFT ARXITEKTURASINI RIVOJLANTIRISH VA INNOVATSION LOYIHALAR KONSEPSIYALARNI QO 'LLASH. ARXITEKTURA, MUHANDISLIK VA ZAMONAVIY TEXNOLOGIYALAR JURNALI, 2(3), 30-34.

11. Abdirasulovna, M. N. (2024). METRIC PROBLEMS IN THE DESIGN OF COATING SHELLS. Synergy: Cross-Disciplinary Journal of Digital Investigation (2995-4827), 2(4), 106-110.

12. Abdirasulovna, M. N. (2023). Samarkand State University of Architecture and Construction. Multidisciplinary Journal of Science and Technology, 3(3), 398-400.

13. Salomovich, T. E., Samariddinovich, S. U., & Pulatovich, M. B. (2023). Improving the heat preservation properties of the exterior walls of brick buildings. International Journal of Culture and Modernity, 28, 15-20.