AMERICAN JOURNAL OF MULTIDISCIPLINARY BULLETIN
ISSN: 2996-511X (online) | ResearchBib (IF) = 9.512 IMPACT FACTOR
Volume-3| Issue-11| 2025 Published: |30-12-2025|

TEYLOR FORMULASINI BA'ZI TADBIQLARI.

https://doi.org/10.5281/zenodo.18018860

Tojiyev G'ayrat Nematovich
Navoiy viloyati Pedagogik mahorat markazi katta o'qituvchisi
Tel:+998939663556 telegramm:@Tojiyev_G_N

Anotatsiya

Mazkur maqolada matematik analizning muhim vositalaridan biri hisoblangan Teylor formulasi va uning ayrim amaliy tadbiqlari yoritilgan. Teylor formulasi yordamida funksiyalarni yaqinlashuvlar orqali ifodalash, murakkab funksiyalarni soddalashtirib hisoblash hamda matematik modellashtirish jarayonlarida qo'llash imkoniyatlari tahlil qilinadi. Maqolada Teylor qatorining asosiy xossalari, qoldiq hadning baholanishi hamda formuladan foydalanish orqali sonli hisoblashlarda aniqlikni oshirish masalalari ko'rib chiqilgan. Shuningdek, fizika, texnika va iqtisodiy masalalarda uchraydigan ayrim misollar asosida Teylor formulasining amaliy ahamiyati ochib berilgan. Tadqiqot natijalari Teylor formulasini o'rganish va uni qo'llash metodikasini takomillashtirishga xizmat qiladi.

Tayanch so'zlar

Teylor formulasi, Makleron formulasi, Teylorining lokal formulasi, Teylor qatori, yaqinlashuv, qoldiq had, matematik analiz, amaliy tadbiqlar.

Аннотация

В данной статье рассматривается формула Тейлора, считающаяся одним из важных инструментов математического анализа, и некоторые её практические применения. Анализируются возможности использования формулы Тейлора для выражения функций через приближения, упрощения комплексных функций и её применения в процессах математического моделирования. В статье рассматриваются основные свойства ряда Тейлора, оценка невязкого члена и вопросы повышения точности численных вычислений с использованием формулы. Также раскрывается практическое значение формулы Тейлора на примерах из физики, техники и экономики. Результаты исследования служат для совершенствования методологии изучения формулы Тейлора и её применения.

Ключевые слова

AMERICAN JOURNAL OF MULTIDISCIPLINARY BULLETIN
ISSN: 2996-511X (online) | ResearchBib (IF) = 9.512 IMPACT FACTOR
Volume-3| Issue-11| 2025 Published: |30-12-2025|

формула Тейлора, формула Макларена, локальная формула Тейлора, ряд Тейлора, приближение, остаточный член, математический анализ, практическое применение.

Abstract

This article discusses the Taylor formula, which is considered one of the important tools of mathematical analysis, and some of its practical applications. The possibilities of using the Taylor formula to express functions through approximations, simplify complex functions, and use it in mathematical modeling processes are analyzed. The article considers the main properties of the Taylor series, the evaluation of the residual term, and the issues of increasing the accuracy of numerical calculations using the formula. Also, the practical significance of the Taylor formula is revealed based on some examples found in physics, engineering, and economics. The results of the research serve to improve the methodology for studying the Taylor formula and its application.

Keywords

Taylor formula, McLaren formula, Taylor's local formula, Taylor series, approximation, residual term, mathematical analysis, practical applications.

Hozirgi kunda matematik analiz fanining rivoji va uning turli sohalardagi amaliy ahamiyati tobora ortib bormoqda. Ayniqsa, funksiyalarni tadqiq etish, ularning xossalarini aniqlash hamda murakkab hisoblashlarni soddalashtirishda yaqinlashuv usullarining oʻrni beqiyosdir. Shunday samarali usullardan biri Teylor formulasi boʻlib, u differensiallanuvchi funksiyalarni koʻphadlar yordamida ifodalash imkonini beradi.

Teylor formulasi funksiyaning maʼlum bir nuqta atrofidagi xulq-atvorini tahlil qilish, taxminiy qiymatlarni aniqlash va hisoblash jarayonlarida yuzaga keladigan xatoliklarni baholashda keng qoʻllaniladi. Ushbu formula nafaqat sof matematik masalalarda, balki fizika, mexanika, elektrotexnika, iqtisodiyot va boshqa amaliy fanlarda ham muhim nazariy asos boʻlib xizmat qiladi. Ayniqsa, murakkab funksiyalarni soddalashtirish va sonli usullar orqali hisoblash jarayonlarida Teylor qatoridan foydalanish katta qulaylik yaratadi.

Mazkur maqolaning maqsadi Teylor formulasining nazariy asoslarini qisqacha yoritish hamda uning ayrim amaliy tadbiqlarini misollar orqali koʻrsatib berishdan iborat. Tadqiqot davomida Teylor formulasidan foydalanishning samaradorligi va uning matematik masalalarni yechishdagi ahamiyati tahlil qilinadi.

Teylor formulasi funksiyalarni darajali qatorlar orqali ifodalash, ya'ni murakkab funksiyalarni oddiy ko‘phadlar yordamida taqriban hisoblash, differensial tenglamalarni yechish, maxsus funksiyalarning (trigonometrik, ko‘rsatkichli) qiymatlarini aniq hisoblash va fizik hodisalarni modellashtirishda (masalan, tebranishlar, o‘zgarishlar) keng qo‘llaniladi.

Agar $f(x)$ funksiya $[a,b]$ segmentda $n-1$ tartibli uzluksiz hosilalarga ega bo‘lib $[a,b]$ segmentning har bir ichki nuqtasida chekli n tartibli hosilaga ega bo‘lsa, u vaqtda $\forall x\epsilon[a,b]$ uchun quyidagi teylor formulasi deb ataladigan formula o‘rinldir.

$$f(x)=f(a)+f(a)'(x-a)+\frac{f''(a)(x-a)^2}{2!}+\frac{f'''(a)(x-a)^3}{3!}+\cdots+\frac{f^{(n-1)}(a)(x-a)^{(n-1)}}{(n-1)!}+\frac{f^{(n)}(\xi)(x-a)^{(n)}}{n!}$$

Bunda $\xi=a+\theta(x-a), 0<\theta<1$ agar ushbu formulada a=0 desak quyidagi Makleron forlumasi hosil bo‘ladi

$$f(x)=f(0)+f(0)'(x)+\frac{f''(0)(x)^2}{2!}+\frac{f'''(0)(x)^3}{3!}+\cdots+\frac{f^{(n-1)}(0)(x)^{(n-1)}}{(n-1)!}+\frac{f^{(n)}(\xi)(x)^n}{n!};$$

$\xi=\theta x, 0<\theta<1$ Teylor formulasidagi oxirgi had Teylor formulasining Lagranj ko'rinishidagi qoldiq hadi $R_n(x)$ bilan belgilanadi: $R_n(x)=\frac{f^{(n)}[a+\theta(x-a)](x-a)^{(n)}}{n!}$

Makleron formulasidagi qoldiq had: $R_n(x)=\frac{f^{(n)}(\theta x)(x-a)^{(n)}}{n!}$ Ko`rinishida bo‘ladi.

Biz ushbu maqolada Teylor formulasining ba’zi tatbiqlarini keltiramiz.

1-misol:$cos5^0$ ning qiymatini 10^{-5} aniqlikda taqribiy hisoblang.

Yechillshi: $cosx$ funksiya uchun Makleron formulasini yozamiz:

$cosx=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\cdots+(-1)^n\frac{x^{2n}}{(2n)!}+R_{2n+2}$ ushbu formulada $x=\frac{\pi}{36}$ deb va $\frac{x^2}{2!}=\frac{\pi^2}{2\cdot 36}=0{,}003808$ $\frac{x^4}{4!}=\frac{1}{6}(\frac{x^2}{2})^2=2{,}4\cdot 10^{-6}$ ekanligini hisobga olsak, taqribiy qiymat sifatida $cosx\approx 1-\frac{x^2}{2}$ ni olish mumkin, bunda xatolik

$$|R_4(x)|=\left|cos\theta x\cdot\frac{x^4}{4!}\right|\leq\frac{|x|^4}{4!}<2{,}5\cdot 10^{-6}$$

demak berilgan aniqlikdagi qiymat: $cos5^0=cos\frac{\pi}{36}=1-0{,}00381=0{,}99619$

2-misol: tengsizlikni isbotlang.$x-\frac{x^2}{2}<\ln(1+x)<x \quad x>0$

Yechilishi: $R_2(x)$ qoldiq hadli Makleron formulasiga ko‘ra:

$$\ln(1+x)=x-\frac{x^2}{2(1+\xi)^2} \quad 0<\xi<x \qquad R_3(x)$$

hadli xuddi shu formulaga ko‘ra $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3(1+\xi_1)^3}$ $0 < \xi_1 < x\,, x > 0$ da $\frac{x^2}{2(1+\xi)^2}$ va $\frac{x^3}{3(1+\xi_1)^3} > 0$ bo‘lgani uchun yuqoridagilardan $x - \frac{x^2}{2} < \ln(1+x) < x$ bo‘lishini bilib olamiz

Ko‘p hollarda teylor formulasining Peano ko‘rinishidagi qoldiq hadli ko‘rinish qulaylik tug‘diradi: $f(x) = f(a) + \frac{f(a)'}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots + \frac{f^{(n)}(x)}{(n)!}(x-a)^n + \theta(|(x-a)^n|$ bunda $\theta(|(x-a)^n|$ yozuv uning $x \to a$ da $(|(x-a)^n|$ ga nisbatan yuqori tartibli cheksiz kichik miqdor ekanligini bildiradi, yani $\lim_{x\to 0} \frac{\theta(|(x-a)^n|}{|x-a|^n} = 0$ $Xususan\ a = 0\ da$ $f(x) = f(0) + \frac{f(0)'}{1!}(x) + \frac{f''(0)}{2!}(x)^2 + \frac{f'''(0)}{3!}(x)^3 + \cdots + \frac{f^{(n)}(0)}{(n)!}(x)^n + \theta(|(x)^n|$ Ushbu formula teylorning lokal formulasi ham deyiladi. Ushbu formulani limitlarini hisoblashga tatbiq etamiz:

3- misol: limitni hisoblang. $\lim_{x\to 0} \frac{1-\sqrt{1+x^2}cosx}{tg^4x} = 0$

Yechilishi: $\sqrt{1+x^2}$ $va\ cosx$ funksiyalar uchun teylor formulasi.$(1+x^2)^{\frac{1}{2}} = 1 + \frac{1}{2}x^2 + \frac{\frac{1}{2}(-\frac{1}{2})}{2}x^4 + \theta(x^4)$ $cosx = 1 - \frac{x^2}{22} + \frac{x^2}{24} + \theta(x^5)$ bo‘lganidan

$$\lim_{x\to 0} \frac{1-\sqrt{1+x^2}cosx}{tg^4x}$$

$$= \lim_{x\to 0} \frac{1-(1+x^2)^{\frac{1}{2}}}{x^4}$$

$$= \lim_{x\to 0} \frac{1 - \left[1 + \frac{1}{2}x^2 + \frac{\frac{1}{2}(-\frac{1}{2})}{2}x^4 + \theta(x^4)\right]\left[1 - \frac{x^2}{22} + \frac{x^2}{24} + \theta(x^5)\right]}{x^4}$$

$$= \lim_{x\to 0} \frac{\frac{1}{4}x^4 + \frac{1}{8}x^4 - \frac{1}{24}x^4 + \theta(x^4)}{x^4} = \lim_{x\to 0}(\frac{1}{3} + \frac{\theta(x^4)}{x^4}) = \frac{1}{3}$$

*4-misol:*limitni toping. $\lim_{x\to 0} \frac{e^x - e^{-x} - 2x}{x - sinx}$

Yechilishi: bizga ma’lumki,

$e^x = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \theta(x^3)sinx = x - \frac{x^3}{3!} + \theta(x^3)$ u holda $\lim_{x\to 0} \frac{e^x - e^{-x} - 2x}{x - sinx} = \lim_{x\to 0} \frac{\frac{x^3}{3} + \theta(x^3)}{\frac{x^3}{6} + \theta(x^3)} = \lim_{x\to 0} \frac{\frac{x^3}{2}}{\frac{x^3}{6}} = 0$

*5-misol:*limitni toping. $\lim_{x\to 0}(\frac{1}{x^2} - \frac{1}{sin^2x})$

AMERICAN JOURNAL OF MULTIDISCIPLINARY BULLETIN
ISSN: 2996-511X (online) | ResearchBib (IF) = 9.512 IMPACT FACTOR
Volume-3| Issue-11| 2025 Published: |30-12-2025|

Yechilishi:

$$\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{1}{sin^2x}\right)=\lim_{x\to 0}\frac{sin^2x-x^2}{x^2sin^2x}=\lim_{x\to 0}\frac{(x-\frac{x^3}{3!}+\theta(x^3))^3\left[x-\frac{x^3}{3!}+\theta(x^3)\right]^2-x^2}{x^2[x+\theta(x)]^2}=$$

$$\lim_{x\to 0}\frac{-\frac{x^4}{3!}+\theta(x^4)}{x^2[x+\theta(x)]^2}=\lim_{x\to 0}\frac{-\frac{x^4}{3}+\theta(x^4)}{x^4+\theta(x^4)}=\lim_{x\to 0}\frac{-\frac{x^4}{3}}{x^4}=-\frac{1}{3}$$

*6-misol:*limitni toping $\lim_{x\to 0}(\frac{sinx}{x})^{\frac{1}{x}}$

Yechilishi: $lny=\lim_{x\to 0}\frac{1}{x}\,ln\frac{sinx}{x}=\lim_{x\to 0}\frac{ln\frac{x+\theta(x^2)}{x}}{x}=\lim_{x\to 0}\frac{\ln(1+\theta(x))}{x}=$ $\lim_{x\to 0}\frac{\theta(x)}{x}=0$ demak $\lim_{x\to 0}(\frac{sinx}{x})^{\frac{1}{x}}=1$

7-misol: π *soni hisoblang*.

Yechilishi: $arctgx=x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\cdots$ $(-1\le x\le 1)$ *tenglikda* $x=\frac{1}{\sqrt{3}}$ deb olsak quyidagi tenglikka ega bo‘lamiz $\frac{\pi}{6}=\frac{1}{\sqrt{3}}(1-\frac{1}{3}\cdot\frac{1}{3}+\frac{1}{5}\cdot\frac{1}{32}-\frac{1}{7}\cdot\frac{1}{32}+\cdots)$ ekanini topamiz. Oxirgi tenglikdan foydanadigan adaan π sonini istalgan aniqlikda hisoblash mumkin.

FOYDALANILGAN ADABIYOTLAR RO‘YXATI:

1. Jo‘rayev I.M.,Tojiyev G‘.N.,Ibodova D.B.,Yaxshiyev.G‘.E. Buxoro Davlat Universiteti ilmiy axboroti 2014/4(56).-B.: Sharq-Buxoro, 2014.-148b.
2. Azlarov T. A., Mansurov X. T. Matematik analiz. 1-,2-q. **-** T.: O‘qituvchi, 1994. **-** 430 b.
3. Sadullayev X., Mansurov T., Xudayberganov G., Vorisov A. K. Matematik analiz kursidan misol va masalalar to‘plami. 1-, 2**-** q. - T.: O‘qituvchi, 1993,- B. 88-99.
4. **Демидович** B.P. Сборник задач по математическому анализу. -М.: Наука, 1990.-С. 4.
5. Xudayberganov G., Vorisov A. K., Mansurov X. T., Shoimqulov B. A. Matematik analizdan ma’ruzalar. **1-,2-q.** -Toshkent, Voris, **2010.** -352 b.