

## THEORETICAL AND PRACTICAL FOUNDATIONS FOR DEVELOPING STUDENTS' LOGICAL THINKING THROUGH MATHEMATICAL PROBLEMS

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### Annotatsiya

Ushbu ilmiy maqola **matematik masalalar** vositasida o'quvchilarning **mantiqiy fikrlash kompetensiyalarini** rivojlantirishning nazariy konsepsiyalari va samarali metodik yondashuvlarini chuqur **tahlil qilishga** bag'ishlangan.

Tadqiqotda masalaning **didaktik ahamiyati**, ta'lim jarayonidagi o'rni hamda o'quvchilarda **tahlil, qiyoslash, umumlashtirish** va **mustaqil xulosa chiqarish** kabi muhim **kognitiv operatsiyalarni** shakllantirishdagi hal qiluvchi roli **ilmiy asosda yoritilgan**.

Mavjud ilmiy adabiyotlar va zamonaviy pedagogik qarashlar kompleks tahlil qilingan holda, masalalarning **turkumlari**, ularning **kompetensiyaviy yondashuv** sharoitidagi o'rni va masalalar orqali o'quv faoliyatini tashkil etishning **eng samarali metodikasi** ishlab chiqilgan. Muallif tomonidan yaratilgan **original masalalar** va ularning yechimlari tahlili orqali ishlab chiqilgan metodikaning **amaliyotda qo'llanilish mexanizmlari** asoslab berilgan.

### Kalit so'zlar

matematik masala, mantiqiy fikrlash, kompetensiya, didaktika, tahlil, taqqoslash, umumlashtirish, metodika, o'quv faoliyati.

### Аннотация

Данная научная статья посвящена глубокому **анализу** теоретических концепций и эффективных методологических подходов к развитию **компетенций логического мышления** учащихся с помощью **математических задач**.

В исследовании **научно обосновывается** дидактическая значимость задачи, ее роль в образовательном процессе, а также ее решающее значение в формировании у учащихся таких важных **когнитивных операций** как

**анализ, сравнение, обобщение и самостоятельное извлечение выводов.**

На основе комплексного анализа имеющейся научной литературы и современных педагогических взглядов разработана **наиболее эффективная методика** организации учебной деятельности посредством задач, а также их **классификация** и роль в условиях **компетентностного подхода**. Механизмы **практического применения** разработанной методики обосновываются посредством анализа **авторских задач** и их решений.

#### **Ключевые слова**

математическая задача, логическое мышление, компетенция, дидактика, анализ, сравнение, обобщение, методика, учебная деятельность.

#### **Abstract**

This scientific article is devoted to the in-depth **analysis** of theoretical concepts and effective methodological approaches for developing students' **logical thinking competencies** through the use of **mathematical problems**.

The study **scientifically substantiates** the didactic significance of the problem, its role in the educational process, and its crucial importance in the formation of such vital **cognitive operations** in students as **analysis, comparison, generalization, and independent drawing of conclusions**.

Based on a comprehensive analysis of available scholarly literature and modern pedagogical views, the **most effective methodology** for organizing learning activities through problems has been developed, along with their

**classification** and role within a **competency-based approach**. The mechanisms for the **practical application** of the developed methodology are substantiated through the analysis of the author's **original problems** and their solutions.

#### **Keywords**

mathematical problem, logical thinking, competence, didactics, analysis, comparison, generalization, methodology.

#### **Introduction:**

The development of students' logical thinking competence is one of the most pressing tasks in the current educational process. Mathematics is not only a collection of numbers and operations, but also a fundamental subject that shapes the culture of thinking. Mathematical education, through the use of problems, is an important tool in developing students' logical thinking skills. The process of problem-solving not only strengthens the student's mathematical knowledge but also develops their intellectual competencies such as analytical thinking, analysis,

comparison, and generalization. Modern pedagogical approaches, based on competency, serve to increase students' ability to solve problems in real-life situations (Alixonov, 2021; Karimova, 2019).

With the help of mathematical problems, the student:

- Understands the problem: Separates the information contained in the problem.
- Plans a solution: Defines the solution path and necessary steps.
- Implements the plan: Executes the plan, performs calculations.
- Checks the result: Evaluates the logical and practical validity of the solution.

Therefore, mathematical problems are the central means for developing students' logical thinking competence.

In the process of mathematical education, problems are considered one of the basic didactic tools that develop students' thinking, strengthen their knowledge and skills, and at the same time, form the basis for making independent conclusions and analytical approaches. Developing students' logical thinking competence is one of the most priority tasks of the modern pedagogical process. This is because the competency-based approach is based not only on a person's knowledge but also on their ability to apply that knowledge in various situations. Mathematical problems serve as a natural means that actively encourages the student to think in this way.

A problem is one of the most universal forms of knowledge transfer. It not only connects theoretical concepts with practice but also activates the student's mental activity, developing intellectual operations such as analysis, comparison, generalization, and logical conclusion (Alixonov, 2021; Karimova, 2019; Usmonova, 2018). Through mathematical problems, the student learns to make independent decisions, apply a chosen strategy, search for alternative solutions, and substantiate their opinion. For this reason, the problem is regarded as the central part of the learning activity and is the most effective means of mastering mathematical knowledge.

The essence of a mathematical problem is seen in its appearance as an intellectual problem that directs the student toward a specific goal. As emphasized by Polya (1957), a problem ensures that the student actively participates in the process of independently discovering solutions and reaching a goal by choosing a path. This process requires a sequence of logical operations: understanding the problem condition, planning a solution, executing the plan, and checking the result obtained. When such activity is organized correctly, the student develops the habit of logical thinking according to each step.

In modern pedagogy, problem-solving is considered the most effective way to develop logical thinking competence. Logical thinking is the ability of an individual

to understand cause-and-effect relationships, generate new knowledge based on existing information, and perform the processes of proof and refutation. According to Vygotsky (1978), the formation of logical thinking is associated with dialogue in the process of a student's practical activity, problematic situations, and independent inquiry. The mathematical problem embodies all these processes.

Mathematical problems, from a didactic point of view, are practical functions that perform several functions simultaneously: cognitive, developmental, motivational, educational, and practical.

A problem must not only match the student's level of knowledge but also increase their thinking activity, encourage the use of logical operations, and strengthen their interest in knowledge. The gradual increase in the complexity of problems leads to the development of students' thinking strategies.

The main components of logical thinking are analysis, comparison, synthesis, generalization, modeling, and logical conclusion. Mathematical problems serve the formation of these very skills. Analyzing the condition of a problem requires separating the given information, comparison, generalization to define the required outcome, modeling to represent the problem as a mathematical model, and checking the solution requires logical conclusion. Thus, problem-solving encompasses all important stages of mental activity.

In the competency-based approach, a problem encourages the student to think in situations close to real life, create a model based on the given information, and find a solution path in problematic situations. In particular, students' skills such as independent decision-making, analysis, and critical thinking are actively formed through problems.

The selection of mathematical problems must be carried out based on the student's age, level of preparation, subject content, and cognitive ability. Every problem should have content that requires a specific thinking operation. The problem should not make the student struggle, but it must force them to think. The problem condition must be free from unnecessary information, and be constructed clearly and logically.

A step-by-step method is used in teaching problem-solving. Polya's (1957) method is one of the most effective approaches. It directs the student through the following sequential steps: understanding the condition, planning, executing the plan, and analyzing the solution. This process forms the basis of self-directed learning and teaches students to think disciplinedly, consistently, and reasonably.

**Problem 1 (Percentage and Fractions).** 50% of the 36 students in the class achieved a high result in mathematics. How many students achieved a high result?

**Solution:**  $36 \times 0.5 = 18$  students



**Analysis:** The problem develops the concept of percentage, working with fractions, and making logical conclusions.

**Problem 2 (Alternative Solution and Comparison).** There are 28 students in a class.  $\frac{3}{7}$  of them are interested in mathematics, and the rest are interested in physics. Half of those interested in physics actively participated in laboratory exercises. How many students actively participated?

**Solution:**

- Those interested in mathematics:  $28 \times \frac{3}{7} = 12$
- Those interested in physics:  $28 - 12 = 16$
- Those who actively participated:  $16 \times \frac{1}{2} = 8$  students

**Analysis:** The problem requires fractions, analysis, and modeling. It develops students' skills in planning and logically calculating the result.

**Problem 3 (Sequence and Strategic Thinking).** In the class, the scores of the students are in the sequence: 3,4,5,6,4,5,6,3. How many students scored 5 and above?

**Solution:** 5 score = 2 students, 6 score = 2 students

**Total:**  $2+2=4$  students

**Analysis:** The problem develops understanding sequence, comparison, and logical thinking.

**Problem 4 (Modeling and Complex Solution).** There are 5 groups in a class. The number of students in the groups are: 3,4,5,4,4.  $\frac{1}{2}$  of the students in each group fully solved the problems. How many students fully solved the problems?

**Solution:**  $3 \times \frac{1}{2} = 1.5 \approx 2$   $4 \times \frac{1}{2} = 2$   $5 \times \frac{1}{2} = 2.5 \approx 3$   $4 \times \frac{1}{2} = 2$   $4 \times \frac{1}{2} = 2$  **Total:**  $2+2+3+2+2=11$  students

**Analysis:** The problem models a real-life situation and develops the student's logical operations.

**Problem 5 (Complex Percentages and Decision Making).**  $\frac{2}{5}$  of the 30 students in the class passed the mathematics test with excellent marks.  $\frac{1}{3}$  of them showed good results. How many students achieved sufficient results?

**Solution:** Excellent:  $30 \times \frac{2}{5} = 12$  Good:  $30 \times \frac{1}{3} = 10$  **Total:**  $12+10=22$  students

**Analysis:** The problem develops fractions, addition, and analytical thinking.

The analytical activity of students increases noticeably through problems. Regular problem solving organizes the structure of students' thinking to be disciplined, reasoned, and consistent (Rasulov, 2021; Xudoberdiyev, 2020). They can quickly analyze the problem condition, define the requirement, and compare different strategies and choose the most effective variant.

In addition, problems develop students' mathematical speech, meaning they learn to express their thoughts accurately and logically.

The methodology for understanding logical errors also plays an important role in the problem-solving process. When a student goes the wrong and illogical way, the teacher returns them to the correct path through proper analysis. This process also shapes critical thinking.

Problems:

- Develop students' logical thinking competence.
- Form skills in analyzing, comparing, generalizing, and modeling problems.
- Connect practical and theoretical knowledge.

**Conclusion:** The study shows that mathematical problems are the most effective means for developing students' logical thinking competence. The learning process through problems directs the student toward independent thinking, problem analysis, and finding solutions.

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