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TIP O‘ZGARISH CHIZIG‘I SILLIQ BO‘LMAGAN PARABOLO-GIPERBOLIK TENGLAMA UCHUN TO‘RTINCHI NOLOKAL MASALA

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Ushbu ishda tip o‘zgarish chizig‘i silliq bo‘lmagan parabolo-giperbolik tenglama uchun qo‘yilgan integral shartli masalaning bir qiymatli yechilishi tadqiq qilingan.

Ma’lumki, aralash tipdagi differensial tenglamalar bo‘yicha tadqiqotlar tarixi bir asrga yaqin bo‘lib, hozirgi kunda xususiy hosilali differensial tenglamalar nazariyasining jadal rivojlanayotgan yo‘nalishlaridan biri hisoblanadi.

Aralash tipdagi parabolo-giperbolik tenglamalar uchun turli lokal va nolokal shartli masalalar xorijiy va respublikamiz olimlari tomonidan tadqiqotlar olib borishmoqda.

xOy tekisligining $x \geq 0,\ y \geq 0$ bo‘lganda $x=1,\ y=1$ to‘g‘ri chiziqlar bilan, $x \geq 0,\ y \leq 0$ bo‘lganda $x=0,\ x-y=1$ hamda $x \leq 0,\ y \geq 0$ bo‘lganda esa $y=0$, $x-y=-1$ to‘g‘ri chiziqlar bilan chegaralangan D sohada

$$u_{xx}-\frac{1}{2}\left(1-sign\left(xy\right)\right)u_{yy}-\frac{1}{2}\left(1+sign\left(xy\right)\right)u_{y}=0 \qquad (1)$$

tenglamani qaraylik,

$D_0 = D \cap \{(x,y): x>0, y>0\}$, $D_1 = D \cap \{(x,y): y<0, x+y>0\}$,
$D_2 = D \cap \{(x,y): y<0, x+y<0\}$, $D_3 = D \cap \{(x,y): x<0, x+y>0\}$,
$D_4 = D \cap \{(x,y): x<0, x+y<0\}$;

OA_1, OA_2, OB_1, OB_2 lar mos ravishda $x=0$, $y=0$ chiziqlarning kesmalari, bu yerda $O(0,0)$, $A_1(0,1)$, $B_1(1,0)$, $A_2(0,-1)$, $B_2(-1,0)$, $E(1/2,-1/2)$, $F(-1/2,1/2)$, $I=\{(x,y): x+y=0, (-1/2)<x<(1/2)\}$.

(1) tenglama D_0 va $D_1 \cup D_2$ $\left(D_3 \cup D_4\right)$ sohalarda mos ravishda parabolik va giperbolik tipga tegishli bo‘lib, u D_0 sohada

$$u_{xx}-u_{y}=0,\quad (x,y)\in D_0, \qquad (2)$$

$D_1 \cup D_2$ va $D_3 \cup D_4$ sohalarda

$$u_{xx}-u_{yy}=0,\quad (x,y)\in D_1 \cup D_2 \cup D_3 \cup D_4 \qquad (3)$$

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ko'rinishlarda yoziladi; OB_1 va OA_1 - (1) tenglamaning tip o'zgarish chiziqlari bo'lib, OB_1 - (1) tenglama uchun xarakteristika bo'ladi. OA_1 esa xarakteristika bo'lmaydi.

D sohada quyidagi masalani qaraylik.

BS_3 **masala.** Quyidagi shartlarni qanoatlantiruvchi $u(x,y)$ funksiya topilsin:

1) D_0, D_1, D_2, D_3, D_4 sohalarda (1) tenglamaning regulyar yechimi bo'lsin;

2) $u(x,y)\in C(\bar{D})\cap C^{2,1}_{x,y}(D_0)\cap C^{2,2}_{x,y}(D_1\cup D_2\cup D_3\cup D_4\setminus I)$ sinfga tegishli bo'lsin;

3) quyidagi shartlarni qanoatlantirsin:

$$u(1,y)=\alpha(y)\int_0^l u(x,y)dx+\varphi(y), \qquad 0\le y\le 1; \qquad (4)$$

$$u_x(0,y)=f_1(y), \qquad -1<y<0; \qquad (5)$$

$$u(0,y)=-u(0,-y)+f_2(y), \qquad -1\le y\le 0; \qquad (6)$$

$$u_y(x,0)=g_1(x), \qquad -1<x<0; \qquad (7)$$

$$u(x,0)=u(-x,0)+g_2(x), \qquad -1\le x\le 0 \qquad (8)$$

shartlarni, OB_1 va OA_1 tip o'zgarish chiziqlarida esa

$u_y(x,+0)=u_y(x,-0)$, $0<x<1$,

$u_x(+0,y)=u_x(x,-0)$, $0<y<1$

ulash shartlarini bajarsin, bu yerda $\alpha(y),\varphi(y),f_1(y),f_2(y),g_1(x),g_2(x)$ - berilgan uzluksiz funksiyalar, $\alpha(0)\ne 2$.

Masalani tadqiq qilishda quyidagi belgilashlardan foydalanamiz:

$u(x,0)=\tau_1(x)$, $0\le x\le 1$; $u_y(x,0)=\nu_1(x)$, $0<x<1$; $u(0,y)=\tau_2(y)$, $0\le y\le 1$; $u_x(0,y)=\nu_2(y)$, $0<y<1$; $\tau_j(x)\in C[0,1]\cap C^2(0,1)$, $\nu_j(x)\in C^1(0,1)\cap L(0,1)$, $j=\overline{1,4}$.

U holda $u(x,y)$ funksiyani D_j $j=\overline{1,4}$ sohalarda (1) tenglama uchun Koshi masalasining yechimi sifatida quyidagicha yozish mumkin:

$$u(x,y)=\frac{1}{2}[\tau_1(x+y)+\tau_1(x-y)]+\frac{1}{2}\int_{x-y}^{x+y}\nu_1(t)dt, \qquad (x,y)\in D_1; \qquad (9)$$

$$u(x,y)=\frac{1}{2}[\tau_2^*(y+x)+\tau_2^*(y-x)]+\frac{1}{2}\int_{y-x}^{y+x}\nu_2^*(t)dt, \qquad (x,y)\in D_2; \qquad (10)$$

$$u(x,y)=\frac{1}{2}[\tau_2(y+x)+\tau_2(y-x)]+\frac{1}{2}\int_{y-x}^{y+x}\nu_2(t)\,dt, \qquad (x,y)\in D_3; \qquad (11)$$

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$$u(x,y)=\frac{1}{2}\left[\tau_1^*(x+y)+\tau_1^*(x-y)\right]+\frac{1}{2}\int_{x-y}^{x+y}\nu_1^*(t)\,dt, \qquad (x,y)\in D_4. \qquad (12)$$

Masalaning qo‘yilishiga ko‘ra

$$\lim_{y\to -x-0}u(x,y)=\lim_{y\to -x+0}u(x,y), \quad 0\le x\le (1/2) \qquad (13)$$

tenglik o‘rinli.

(9) va (10) formulalarni (13) shartga bo‘ysundirib,

$$\tau_1(0)+\tau_1(2x)-\int_0^{2x}\nu_1(t)\,dt=\tau_2^*(0)+\tau_2^*(-2x)+\int_{-2x}^{0}\nu_2^*(t)\,dt, \quad 0\le x\le 1/2$$

munosabatga ega bo‘lamiz.

Oxirgi tenglikda $\tau_1(0)=\tau_2^*(0)$ ekanligini tenglikni e’tiborga olib hamda $2x=z$ almashirish bajarasak,

$$\tau_1(z)-\tau_2^*(-z)=\int_0^z\left[\nu_1(t)+\nu_2^*(-t)\right]dt, \quad 0\le z\le 1 \qquad (14)$$

ko‘rinishda yoziladi.

(14) tenglikda (5), (6) shartlarni va yuqoridagi belgilashlarni e’tiborga olib quyidagini topamiz:

$$\tau_1(z)+\tau_2(z)=\int_0^z\left[\nu_1(t)+f_1(-t)\right]dt+f_2(-z). \qquad (15)$$

Bunda $x=z$ almashirish bajarib

$$\tau_1(x)+\tau_2(x)=\int_0^x\left[\nu_1(t)+f_1(-t)\right]dt+f_2(-x), \quad 0\le x\le 1 \qquad (16)$$

ega bo‘lamiz. Endi

$$\lim_{x\to -y-0}u(x,y)=\lim_{x\to -y+0}u(x,y), \quad 0\le y\le 1/2 \qquad (17)$$

munosabatga (11) va (12) formulalarni bo‘ysundirsak, ushbu

$$\tau_2(0)+\tau_2(2y)-\int_0^{2y}\nu_1(t)\,dt=\tau_1^*(0)+\tau_1^*(-2y)+\int_{-2y}^{0}\nu_1^*(t)\,dt, \quad 0\le y\le 1/2$$

tenglikni topamiz.

Oxirgi tenglikda $\tau_2(0)=\tau_1^*(0)$ ekanligini e’tiborga olib hamda $2y=z$ almashirish bajarsak

$$\tau_2(z)-\tau_1^*(-z)=\int_0^z\left[\nu_2(t)+\nu_1^*(-t)\right]dt, \quad 0\le z\le 1 \qquad (18)$$

munosabatga ega bo‘lamiz.

(18) tenglikda (7), (8) shartlarni va yuqoridagi belgilashlarni e’tiborga olib,

$$\tau_1(z)+\tau_2(z)=\int_0^z[\nu_2(t)+g_1(-t)]dt+g_2(-z) \qquad (19)$$

ifodani topamiz.

Bunda $y=z$ almashirish bajarsak

$$\tau_1(y)+\tau_2(y)=\int_0^y[\nu_2(t)+g_1(-t)]dt+g_2(-y) \qquad (20)$$

munosabatga ega boʻlamiz.

Endi (15) va (19) tengliklardan oʻng tomonlarini tenglab

$$\int_0^z[\nu_1(t)+f_1(-t)]dt+f_2(-z)=\int_0^z[\nu_2(t)+g_1(-t)]dt+g_2(-z)$$

$$\nu_1(z)+f_1(-z)+f_2'(-z)=\nu_2(z)+g_1(-z)+g_2'(-z)$$

$$\nu_1(z)-\nu_2(z)=g_1(-z)-f_1(-z)+g_2'(-z)-f_2'(-z)$$

$$\nu_1(x)-\nu_2(x)=g_1(-x)-f_1(-x)+g_2'(-x)-f_2'(-x), \qquad 0<x<1 \qquad (21)$$

topamiz.

(2) tenglama va (4), (6) shartlarda y ni nolga intiltirib,

$$\tau_1''(x)=\nu_1(x), \;\; 0<x<1; \;\; \tau_1(0)=f_2(0)/2, \;\; \tau_1(1)=\alpha(0)\int_0^1\tau_1(x)dx+\varphi(0) \qquad (22)$$

bunda chegaraviy masalaga ega boʻlamiz.

$$z(x)=\tau_1(x)+\tau_1(0)(x-1)-\tau_1(1)x$$

almashtirish bajarsak

$$z''(x)=\tau_1''(x)=\nu_1(x) \quad z(0)=z(1)=0$$

bir jinsli masala hosil boʻladi. Oddiy differensial tenglamalar kursidan ma’lumki, Grin funksiyasi usuliga koʻra bu massalaning yechimi

$$z(x)=\int_0^1 G(x,t)\nu_1(t)dt \qquad (23)$$

koʻrinishda boʻladi. Bu masala yagona yechimga ega:

$$\tau_1(x)=\tau_1(0)(1-x)+\tau_1(1)x+\int_0^1 G(x,t)\nu_1(t)dt$$

$$\int_0^1\tau_1(x)dx=\int_0^1\left(\tau_1(0)(1-x)+\tau_1(1)x+\int_0^1 G(x,t)\nu_1(t)dt\right)dx$$

$$\int_0^1\tau_1(x)dx=\int_0^1\tau_1(0)(1-x)dx+\int_0^1\tau_1(1)xdx+\int_0^1\int_0^1 G(x,t)\nu_1(t)dtdx$$

$$\int_0^1\tau_1(x)dx=\tau_1(0)\int_0^1(1-x)dx+\tau_1(1)\int_0^1 xdx+\int_0^1\nu_1(t)dt\int_0^1 G(x,t)dx$$

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$$\int_0^1 \tau_1(x)dx = -\tau_1(0)\left.\frac{(1-x)^2}{2}\right|_0^1 + \tau_1(1)\left.\frac{x^2}{2}\right|_0^1 + \int_0^1 \nu_1(t)dt\int_0^1 G(x,t)dx$$

$$\int_0^1 \tau_1(x)dx = -\tau_1(0)\frac{(1-1)^2}{2} + \tau_1(0)\frac{(1-0)^2}{2} + \tau_1(1)\frac{1^2}{2} - \tau_1(1)\frac{0^2}{2} + \int_0^1 \nu_1(t)dt\int_0^1 G(x,t)dx$$

$$\int_0^1 \tau_1(x)dx = \tau_1(0)\frac{1}{2} + \tau_1(1)\frac{1}{2} + \int_0^1 \nu_1(t)dt\int_0^1 G(x,t)dx$$

$$\tau_1(1) = \alpha(0)\left(\tau_1(0)\frac{1}{2} + \tau_1(1)\frac{1}{2} + \int_0^1 \nu_1(t)dt\int_0^1 G(x,t)dx\right) + \varphi(0)$$

$$\tau_1(1) = \alpha(0)\tau_1(0)\frac{1}{2} + \alpha(0)\tau_1(1)\frac{1}{2} + \alpha(0)\int_0^1 \nu_1(t)dt\int_0^1 G(x,t)dx + \varphi(0)$$

$$\left(2-\alpha(0)\right)\tau_1(1) = 2\alpha(0)\tau_1(0) + 2\alpha(0)\int_0^1 \nu_1(t)dt\int_0^1 G(x,t)dx + 2\varphi(0)$$

$$\tau_1(1) = \frac{2\alpha(0)}{\left(2-\alpha(0)\right)}\tau_1(0) + \frac{2\varphi(0)}{\left(2-\alpha(0)\right)} + \frac{2\alpha(0)}{\left(2-\alpha(0)\right)}\int_0^1 \nu_1(t)dt\int_0^1 G(x,t)dx$$

$$\tau_1(0) = \frac{f_2(0)}{2}$$

$$\tau_1(1) = \frac{\alpha(0)f_2(0) + 2\varphi(0)}{\left(2-\alpha(0)\right)} + \frac{2\alpha(0)}{\left(2-\alpha(0)\right)}\int_0^1 \nu_1(t)dt\int_0^1 G(x,t)dx$$

$$\tau_1(x) = \frac{f_2(0)}{2}(1-x) + \left(\frac{\alpha(0)f_2(0) + 2\varphi(0)}{\left(2-\alpha(0)\right)} + \frac{2\alpha(0)}{\left(2-\alpha(0)\right)}\int_0^1 \nu_1(t)dt\int_0^1 G(x,t)dx\right)x + \int_0^1 G(x,t)\nu_1(t)dt$$

$$\tau_1(x) = \frac{f_2(0)}{2} - \frac{f_2(0)}{2}x + \frac{\alpha(0)f_2(0) + 2\varphi(0)}{\left(2-\alpha(0)\right)}x + \frac{2\alpha(0)x}{\left(2-\alpha(0)\right)}\int_0^1 \nu_1(t)dt\int_0^1 G(x,t)dx + \int_0^1 G(x,t)\nu_1(t)dt$$

$$\tau_1(x) = \frac{f_2(0)}{2} + \frac{3\alpha(0)f_2(0) - 2f_2(0) + 4\varphi(0)}{2\left(2-\alpha(0)\right)}x + \int_0^1 \nu_1(t)\left(\frac{2\alpha(0)x}{\left(2-\alpha(0)\right)}\int_0^1 G(x,t)dx\right)dt + \int_0^1 G(x,t)\nu_1(t)dt$$

$$\tau_1(x) = \frac{f_2(0)}{2} + \frac{\left(3/2\,\alpha(0)-1\right)f_2(0) + 2\varphi(0)}{\left(2-\alpha(0)\right)}x + \int_0^1 \nu_1(t)\left(G(x,t) + \frac{2\alpha(0)x}{\left(2-\alpha(0)\right)}\int_0^1 G(x,t)dx\right)dt$$

$$\tau_1(x) = \int_0^1 K(x,t)\nu_1(t)dt + \omega_1(x), \qquad 0\le x\le 1, \qquad (24)$$

bu yerda

$$K(x,t) = G(x,t) + \frac{2\alpha(0)x}{\left(2-\alpha(0)\right)}\int_0^1 G(x,t)dx$$

$$G(x,t)=\begin{cases}(t-1)x, & 0<x<t,\\ (x-1)t, & t<x<1,\end{cases}$$

$$\omega_1(x)=\frac{f_2(0)}{2}+\frac{\left(3/2\alpha(0)-1\right)f_2(0)+2\varphi(0)}{\left(2-\alpha(0)\right)}x \tag{25}$$

Demak, qo‘yilgan masala D_0 sohada (2) tenglamani umumiy yechimidan

$$u(x,y)=\int_0^y \tau_2(\eta)G_{1\xi}(x,y;0,\eta)d\eta-\int_0^y F(\eta)G_{1\xi}(x,y;1,\eta)d\eta+\int_0^1 \tau_1(\xi)G_1(x,y;\xi,0)d\xi \tag{26}$$

ushbu tenglikka ega bo‘lamiz.

Bu yerda (4) shartga ko‘ra shartga ko’ra $F(\eta)=u(1,\eta)=u(0,\eta)$ yoki $F(\eta)=\tau_2(\eta)$ ga teng ekanligini e’tiborga olsak (26) ni x bo‘yicha differensial, so‘ngra $x\to 0$ da limitga o‘tsak,

$$u(x,y)=\int_0^y \tau_2(\eta)G_{1\xi}(x,y;0,\eta)d\eta-\int_0^y \tau_2(\eta)G_{1\xi}(x,y;1,\eta)d\eta+\int_0^1 \tau_1(\xi)G_1(x,y;\xi,0)d\xi$$

$$u_x(x,y)=\int_0^y \tau_2(\eta)G_{1\xi x}(x,y;0,\eta)d\eta-\int_0^y \tau_2(\eta)G_{1\xi x}(x,y;1,\eta)d\eta+\int_0^1 \tau_1(\xi)G_{1x}(x,y;\xi,0)d\xi$$

$$u_x(0,y)=\int_0^y \tau_2(\eta)G_{1\xi x}(0,y;0,\eta)d\eta-\int_0^y \tau_2(\eta)G_{1\xi x}(0,y;1,\eta)d\eta+\int_0^1 \tau_1(\xi)G_{1x}(0,y;\xi,0)d\xi$$

$$\nu_2(y)=\int_0^y \tau_2(\eta)G_{1\xi x}(0,y;0,\eta)d\eta-\int_0^y \tau_2(\eta)G_{1\xi x}(0,y;1,\eta)d\eta+\int_0^1 \tau_1(\xi)G_{1x}(0,y;\xi,0)d\xi$$

munosabatga ega bo‘lamiz, oxirgi tenglikda $G_{1\xi x}=G_{2\eta}$, $G_{1x}=-G_{2\xi}$ ekanligini e’tiborga olsak

$$\nu_2(y)=\int_0^y \tau_2(\eta)G_{2\eta}(0,y;0,\eta)d\eta-\int_0^y \tau_2(\eta)G_{2\eta}(0,y;1,\eta)d\eta+\int_0^1 \tau_1(\xi)G_{2\xi}(0,y;\xi,0)d\xi \tag{27}$$

bo‘ladi, bu yerda

$$G_1(x,y;\xi,\eta)=\frac{1}{2\sqrt{\pi(t-\eta)}}\sum_{n=-\infty}^{+\infty}\left\{\exp\left[-\frac{(x-\xi-2n)^2}{4(t-\eta)}\right]-\exp\left[-\frac{(x+\xi-2n)^2}{4(t-\eta)}\right]\right\}$$

$$G_2(x,y;\xi,\eta)=\frac{1}{2\sqrt{\pi(t-\eta)}}\sum_{n=-\infty}^{+\infty}\left\{\exp\left[-\frac{(x-\xi-2n)^2}{4(t-\eta)}\right]+\exp\left[-\frac{(x+\xi-2n)^2}{4(t-\eta)}\right]\right\}$$

Bunda (16), (21) va (24) munosabatlardan ushbu

$$\nu_1(x)=\nu_2(x)+g_1(-x)-f_1(-x)+g_2'(-x)-f_2'(-x)$$

$$\nu_1(x)=\nu_2(x)+\omega_2(x) \tag{28}$$

$$\tau_1(x)=\int_0^1 K(x,t)\nu_1(t)\,dt+\omega_1(x)$$

$$\tau_1(x)=\int_0^1 K(x,t)\left[\nu_2(t)+\omega_2(t)\right]dt+\omega_1(x)$$

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$$\tau_1(x)=\int_0^1 K(x,t)\nu_2(t)\,dt+\int_0^1 K(x,t)\omega_2(t)dt+\omega_1(x)$$

$$\tau_1(x)=\int_0^1 K(x,t)\nu_2(t)\,dt+\omega_3(x) \qquad (29)$$

$$\tau_1(x)-\tau_2(x)=\int_0^x \nu_2(t)dt+\int_0^x f_1(-t)dt-\int_0^x f_1(-t)dt+\int_0^x g_1(-t)dt+\int_0^x g_2'(-t)dt-\int_0^x f_2'(-t)dt+f_2(-x)$$

$$\tau_2(x)=\tau_1(x)-\int_0^x \nu_2(t)dt+\int_0^x f_1(-t)dt-\int_0^x f_1(-t)dt-\int_0^x g_1(-t)dt-\int_0^x g_2'(-t)dt+\int_0^x f_2'(-t)dt-f_2(-x)$$

$$\tau_2(x)=\tau_1(x)-\int_0^x \nu_2(t)dt-\int_0^x g_1(-t)dt-\int_0^x g_2'(-t)dt-f_2(0)$$

$$\tau_2(x)=\tau_1(x)-\int_0^x \nu_2(t)dt+\omega_4(x)$$

$$\tau_2(x)=\int_0^1 K(x,t)\nu_2(t)\,dt+\omega_3(x)-\int_0^x \nu_2(t)dt+\omega_4(x)$$

$$\tau_2(x)=\int_0^1 K(x,t)\nu_2(t)dt-\int_0^x \nu_2(t)dt+\omega_5(x) \qquad (30)$$

Munosabatlar kelib chiqadi, bu yerda $\omega_2(x)=g_1(-x)-f_1(-x)+g_2'(-x)-f_2'(-x)$ $\omega_3(x)=\int_0^1 K(x,t)\omega_2(t)dt+\omega_1(x)$ $\omega_4(x)=-\int_0^x g_1(-t)dt-\int_0^x g_2'(-t)dt-f_2(0)$ $\omega_5(x)=\omega_4(x)+\omega_3(x)$

Endi (27) tenglikni o'ng tarafini bo'laklab integrallab so'ngra yuqoridagi munosabatlardan foydalansak

$$\nu_2(y)=\int_0^y \tau_2'(\eta)G_2(0,y;0,\eta)d\eta-\int_0^y \tau_2'(\eta)G_2(0,y;1,\eta)d\eta+\int_0^1 \tau_1'(\xi)G_2(0,y;\xi,0)d\xi\eta$$

$$\nu_2(y)=\int_0^y \tau_2'(\eta)\big(G_2(0,y;0,\eta)-G_2(0,y;1,\eta)\big)d\eta+\int_0^1 \tau_1'(\xi)G_2(0,y;\xi,0)d\xi\eta$$

$$\tau_1'(x)=\int_0^1 K_x(x,t)\nu_2(t)\,dt+\omega_3'(x)$$

$$\tau_2'(x)=\int_0^1 K_x(x,t)\nu_2(t)dt-\nu_2(x)+\omega_5'(x)$$

$$\nu_2(y)=\int_0^y\left(\int_0^1 K_\eta(\eta,t)\nu_2(t)dt-\nu_2(\eta)+\omega_5'(\eta)\right)\big(G_2(0,y;0,\eta)-G_2(0,y;1,\eta)\big)d\eta+$$

$$+\int_0^1\left(\int_0^1 K_\xi(\xi,t)\nu_2(t)\,dt+\omega_3'(\xi)\right)G_2(0,y;\xi,0)d\xi$$

$$\nu_2(y)=\int_0^y\int_0^1 K_\eta(\eta,t)\nu_2(t)dt\left(G_2(0,y;0,\eta)-G_2(0,y;1,\eta)\right)d\eta-\int_0^y\nu_2(\eta)\left(G_2(0,y;0,\eta)-G_2(0,y;1,\eta)\right)d\eta+$$

$$+\int_0^y\omega_5'(\eta)\left(G_2(0,y;0,\eta)-G_2(0,y;1,\eta)\right)d\eta+\int_0^1\int_0^1 K_\xi(\xi,t)\nu_2(t)\,dtG_2(0,y;\xi,0)d\xi+\int_0^1\omega_3'(\xi)G_2(0,y;\xi,0)d\xi$$

$$\nu_2(y)=\int_0^y\int_0^1 K_\eta(\eta,t)\nu_2(t)dt\left(G_2(0,y;0,\eta)-G_2(0,y;1,\eta)\right)d\eta-\int_0^y\nu_2(\eta)\left(G_2(0,y;0,\eta)-G_2(0,y;1,\eta)\right)d\eta+$$

$$+\int_0^1\int_0^1 K_\xi(\xi,t)\nu_2(t)\,dtG_2(0,y;\xi,0)d\xi+\int_0^y\omega_5'(\eta)\left(G_2(0,y;0,\eta)-G_2(0,y;1,\eta)\right)d\eta+\int_0^1\omega_3'(\xi)G_2(0,y;\xi,0)d\xi$$

$$\nu_2(y)=\int_0^y\int_0^1 K_\eta(\eta,t)\nu_2(t)dt\left(G_2(0,y;0,\eta)-G_2(0,y;1,\eta)\right)d\eta-\int_0^y\nu_2(\eta)\left(G_2(0,y;0,\eta)-G_2(0,y;1,\eta)\right)d\eta+$$

$$+\int_0^1\int_0^1 K_\xi(\xi,t)\nu_2(t)\,dtG_2(0,y;\xi,0)d\xi+\omega_6(y)$$

(31)

hosil bo'ladi.

bu yerda $\omega_6(y)=\int_0^y\omega_5'(\eta)\left(G_2(0,y;0,\eta)-G_2(0,y;1,\eta)\right)d\eta+\int_0^1\omega_3'(\xi)G_2(0,y;\xi,0)d\xi$

(31) tenglikda integralash tartibini almashtirib, ba'zi hisoblashlarni bajarib, $\nu_2(y)$ noma'lum funksiyaga nisbatan 2-tur Fredgolim integral tenglamasiga ega bo'lamiz:

$$\nu_2(y)=\int_0^1\nu_2(t)\int_0^y K_\eta(\eta,t)\left(G_2(0,y;0,\eta)-G_2(0,y;1,\eta)\right)d\eta dt-\int_0^y\nu_2(\eta)\left(G_2(0,y;0,\eta)-G_2(0,y;1,\eta)\right)d\eta+$$

$$+\int_0^1\nu_2(t)\int_0^1 K_\xi(\xi,t)G_2(0,y;\xi,0)d\xi dt+\omega_6(y)$$

$$\nu_2(y)=\int_0^1\nu_2(t)\left(\int_0^y K_\eta(\eta,t)\left(G_2(0,y;0,\eta)-G_2(0,y;1,\eta)\right)d\eta+\int_0^1 K_\xi(\xi,t)G_2(0,y;\xi,0)d\xi\right)dt-$$

$$-\int_0^y\nu_2(\eta)\left(G_2(0,y;0,\eta)-G_2(0,y;1,\eta)\right)d\eta+\omega_6(y)$$

$$\nu_2(y)=\int_0^1\nu_2(t)\left(\int_0^y K_\eta(\eta,t)\left(G_2(0,y;0,\eta)-G_2(0,y;1,\eta)\right)d\eta+\int_0^1 K_\xi(\xi,t)G_2(0,y;\xi,0)d\xi\right)dt-$$

$$-\int_0^y\nu_2(t)\left(G_2(0,y;0,t)-G_2(0,y;1,t)\right)dt+\omega_6(y)$$

$$\nu_2(y)=\int_0^1\nu_2(t)K_1(y,t)dt+\int_0^y\nu_2(t)K_2(y,t)dt+\omega_6(y)$$

$$\nu_2(y)=\int_y^1 \nu_2(t)K_1(y,t)dt+\int_0^y \nu_2(t)K_1(y,t)dt+\int_0^y \nu_2(t)K_2(y,t)dt+\omega_6(y)$$

$$\nu_2(y)=\int_y^1 \nu_2(t)K_1(y,t)dt+\int_0^y \nu_2(t)\left[K_1(y,t)+K_2(y,t)\right]dt+\omega_6(y)$$

$$\nu_2(y)=\int_0^1 \nu_2(t)K_3(y,t)dt+\omega_6(y), \qquad (32)$$

$$K_3(y,t)=\begin{cases} K_1(y,t)+K_2(y,t), & 0<t<y \\ K_1(y,t), & y<t<1 \end{cases}$$

Bu yerda

$$K_1(y,t)=\int_0^y K_\eta(\eta,t)\left(G_2(0,y;0,\eta)-G_2(0,y;1,\eta)\right)d\eta+\int_0^1 K_\xi(\xi,t)G_2(0,y;\xi,0)d\xi$$

$$K_2(y,t)=G_2(0,y;0,t)-G_2(0,y;1,t)$$

Integral tenglamaning yadrosi $K_3(y,t)$

Masala yechimining yagonaligini ekstremum prinsipi yordamida isbotlaymiz. Buning uchun bir jinsli nolokal masalani qaraymiz, ya'ni

$$\varphi(y)\equiv f_1(y)\equiv f_2(y)\equiv g_1(x)\equiv g_2(x)\equiv 0. \qquad (33)$$

1-teorema. Agar (33) shart bajarilsa, u holda nolokal masalaning $u(x,y)$ yechimi oʻzining musbat maksimumiga (manfiy minimumiga) yopiq $\bar{D}_0$ sohaning $\overline{OA_1}\cup\overline{A_0B_1}$ chegarasida erishadi.

Parabolik tipdagi tenglamalar uchun ma'lum boʻlgan ekstremum prinsipiga asosan (1) tenglamaning $u(x,y)$ yechimi D_0 sohaning ichki nuqtalarida oʻzining musbat maksimumiga (manfiy minimumiga) erishmaydi. (1) tenglamaning $u(x,y)$ yechimi OB_1 intervalning ichki nuqtalarida oʻzining musbat maksimumiga (manfiy minimumiga) erishmasligini koʻrsatamiz. Buning uchun teskarisini faraz qilamiz. $u(x,y)$ funksiya OB_1 intervalning biron $E(x_0,y)\in OB_1$ nuqtasida oʻzining musbat maksimumiga (manfiy minimumiga) erishsin.

(33) ga koʻra (16), (29) va (30) munosabatlardan

$$\left.\begin{aligned} \tau_1(x_0)+\tau_2(x_0)&=\int_0^{x_0}\nu_1(t)dt \\ \tau_1(x_0)&=\int_0^1 K(x_0,t)\nu_2(t)dt \\ \tau_2(x_0)&=\int_0^1 K(x_0,t)\nu_2(t)dt-\int_0^{x_0}\nu_2(t)dt \end{aligned}\right\} \qquad (34)$$

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tenglikga ega bo'lamiz.

$E(x_0, y)$ nuqtada $\tau_1(x_0) > 0$ $(\tau_1(x_0) < 0)$, $\tau_1'(x_0) = 0$ va (33) tenglikdan

$$\nu_1(x_0) > 0 \ (\nu_1(x_0) < 0) \qquad (35)$$

tengsizlikni olamiz.

Ikkinchi tomondan $E(x_0, y)$ nuqtada $\tau_1''(x_0) < 0$ $(\tau_1''(x_0) > 0)$ shartga ko'ra, $\tau_1''(x) = \nu_1(x)$ tenglikdan quyidagi tengsizlikka ega bo'lamiz:

$$\nu_1(x_0) < 0 \ (\nu_1(x_0) > 0).$$

Bu tengsizlik nolokal masalaning 1) shartiga asosan (33) tengsizlikka ziddir. Demak, $u(x, y)$ funksiya OB_1 intervalning ichki nuqtalarida o'zining musbat maksimumiga (manfiy minimumiga) erishmaydi.

Shunday qilib, nolokal masalaning $u(x, y)$ yechimi o'zining musbat maksimumiga (manfiy minimumiga) yopiq $\bar{D}_0$ sohaning $\overline{OA_1} \cup \overline{A_0B_1}$ chegarasida erishadi.

2-teorema. Agar (33) shart bajarilsa, u holda D sohada nolokal masalaning yechimi yagonadir.

1- teoremaga ko'ra (33) shartni e'tiborga olib, yopiq $\bar{D}_0$ sohada $u(x, y) \equiv 0$ tenglikka ega bo'lamiz. $D_1 \cup D_2 \cup D_3 \cup D_4$ sohada (1) tenglama uchun qo'yilgan Koshi masalasi yechimining yagonaligiga asosan yopiq $\overline{D_1} \cup \overline{D_2} \cup \overline{D_3} \cup \overline{D_4}$ sohada $u(x, y) \equiv 0$ ni hosil qilamiz. Shunday qilib, yopiq $\bar{D}$ sohada $u(x, y) \equiv 0$.

Bundan, D sohada nolokal masala yechimining yagonaligi kelib chiqadi.