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METHODOLOGICAL IMPORTANCE OF SOLVING A SINGLE GEOMETRIC PROBLEM USING DIVERSE METHODS

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Abstract

This article analyzes the process of solving a single non-standard geometric problem using various methods (geometric, trigonometric, coordinate, and vector). It highlights the methodology of developing students' creative thinking and interdisciplinary understanding by solving problems through multiple approaches. In teaching mathematics, merely instructing students in ready-made algorithms is insufficient. Modern pedagogy places a high value on the ability to solve a single problem in several ways. This approach demonstrates the intrinsic connection between different branches of science (algebra, geometry, and trigonometry).

Keywords

Geometric problem, multi-method solution, trigonometry, analytic geometry, creative thinking.

Problem Statement: Given a square ABCD. An interior point E is chosen such that triangle AED is an equilateral (regular) triangle (figure 1). Find the degree measure of angle EBC.

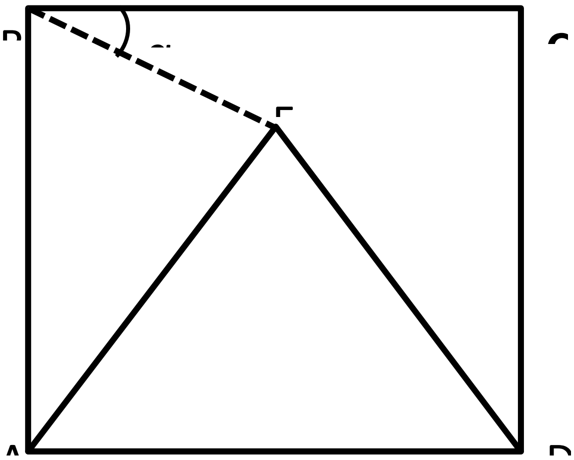

Figure 1.

SOLUTIONS OF THE PROBLEM USING DIFFERENT METHODS

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Let the side of the square be a. Since ABCD is a square: AB = BC = CD = =DA = a and all its angles are equal to 90^0. Since triangle AED is an equilateral triangle: AE = ED = AD = a and all its interior angles are equal to 60^0.

Method 1: Pure Geometric Method (Triangle Congruence and Properties)

Since the angle A of the square is 90^0 and angle EAD of the triangle is 60^0, we find angle EAB = 90^0 - 60^0 =30^0.

Now, let us consider triangle EAB. In this triangle, AB = a and AE = a hence, it is an isosceles triangle with AB = AE. Since the base of the isosceles triangle is EB, its base angles are equal:

$< AEB = <ABE =\frac{180^0-30^0}{2} = 75^0$

The angle B of the square is equal to 90^0. The required angle EBC is found as follows:

$<EBC=90^0-<ABE=90^0-75^0=15^0$

Answer: $<EBC= 15^0$.

Method 2: Trigonometric Method (Theorems and Functions)

Let us draw an altitude EF from point E parallel to the sides AB and CD of the square. According to the formula for the altitude of an equilateral triangle:

$EF = \frac{a\sqrt{3}}{2}$.

Let EG be the distance from point E to the side BC. In this case:

$EG = a - EF = a - \frac{a\sqrt{3}}{2} = a\left(1 - \frac{\sqrt{3}}{2}\right) = \frac{a(2-\sqrt{3})}{2}$

Point G is the midpoint of side BC , which means $BG = \frac{a}{2}$. In the right angled triangle EGB, we apply the tangent function for the $<EBC = \alpha$:

$tg\alpha = \frac{EG}{BG} = \frac{\frac{a(2-\sqrt{3})}{2}}{\frac{a}{2}} = 2 - \sqrt{3}.$

Let us take According to trigonometric tables and formulas, it is known that $tg15^0 = 2 - \sqrt{3}$. Therefore, $\alpha = 15^0$.

Answer: $<EBC= 15^0$.

Method 3: Analytic Geometry (Coordinate Method)

Vertex A of the square as the origin of the coordinate system (0;0). Let side AD lie on the X-axis and side AB lie on the Y-axis. Then, the coordinates of the points will be as follows: *A(0;0), B(0;a), C(a;a), D(a,0)* (figure 2).

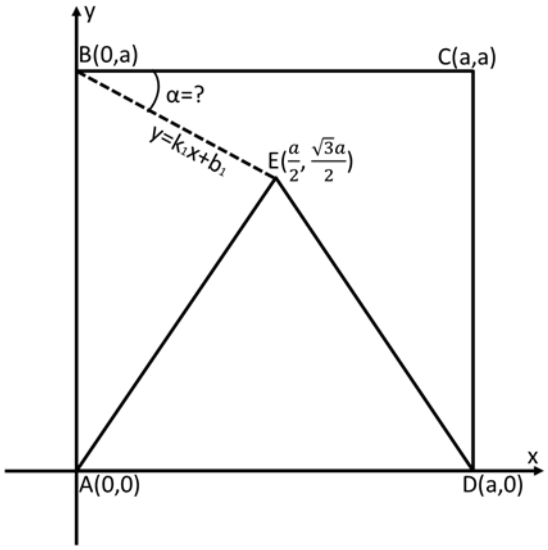

Figure 2.

The coordinates of vertex E of the equilateral triangle will be $E\left(\frac{a}{2};\frac{a\sqrt{3}}{2}\right)$. Let k_1 be the slope (slope coefficient) of the line BE. According to the formula for finding the slope:

$$k_1=\frac{y_E-y_B}{x_E-x_B}=\frac{\frac{a\sqrt{3}}{2}-a}{\frac{a}{2}-0}=\sqrt{3}-2$$

Since the line BC is horizontal, its slope is $k_2=0$. According to the formula for finding the angle between two lines:

$tg\alpha=\left|\frac{k_1-k_2}{1+k_1\cdot k_2}\right|=\left|\frac{\sqrt{3}-2-0}{1+(\sqrt{3}-2)\cdot 0}\right|=2-\sqrt{3}.$

According to trigonometric tables and formulas, it is known that

$tg15^0=2-\sqrt{3}$. Therefore, $\alpha=15^0$.

Answer:<*EBC*= *15*⁰.

Method 4: Analytic Geometry (Vector Method)

This method is of great methodological importance in developing the skills of upper-grade students and undergraduates in applying vector coordinates and the properties of the dot (scalar) product of vectors.

We use the same coordinate system as in Method 3: let *A(0;0)* be the origin. Thus, *B(0;a)*, *C(a;a)*, and $E\left(\frac{a}{2};\frac{a\sqrt{3}}{2}\right)$.

Let us determine the coordinates of the vectors $\overrightarrow{BC}$ and $\overrightarrow{BE}$ that form the unknown angle:

$\overrightarrow{BC}=(x_C-x_B;y_C-y_B)=(a-0;a-a)=(a;0)$

$\overrightarrow{BE}=(x_E-x_B;y_E-y_B)=(\frac{a}{2}-0;\frac{a\sqrt{3}}{2}-a)=\left(\frac{a}{2};a\left(\frac{\sqrt{3}}{2}-1\right)\right)$

Now, we apply the formula for the angle between two vectors:

$$cos\alpha = \frac{\overrightarrow{BC} \cdot \overrightarrow{BE}}{|\overrightarrow{BC}| \cdot |\overrightarrow{BE}|}$$

We calculate the dot product and the magnitudes of the vectors:

$$\overrightarrow{BC} \cdot \overrightarrow{BE} = a \cdot \frac{a}{2} + 0 \cdot a\left(\frac{\sqrt{3}}{2} - 1\right) = \frac{a^2}{2}$$

$|\overrightarrow{BC}| = \sqrt{a^2 + 0^2} = a$

$|\overrightarrow{BE}| = \sqrt{\left(\frac{a}{2}\right)^2 + a^2\left(\frac{\sqrt{3}}{2} - 1\right)^2} = \sqrt{\frac{a^2}{4} + a^2\left(\frac{3}{4} - \sqrt{3} + 1\right)} = a\sqrt{2 - \sqrt{3}}$

We substitute the found values into the formula:

$cos\alpha = \frac{\frac{a^2}{2}}{a \cdot a\sqrt{2-\sqrt{3}}} = \frac{1}{2\sqrt{2-\sqrt{3}}} = \frac{\sqrt{2+\sqrt{3}}}{2}$

According to trigonometric tables and formulas, it is known that

$cos15^0 = \frac{\sqrt{2+\sqrt{3}}}{2}$. Therefore, $\alpha = 15^0$.

Answer: <*EBC*= *15⁰*.

CONCLUSION

Solving a single problem using 4 different methods (geometric, trigonometric, coordinate, and vector) increases interactivity in mathematics classes. The geometric method enhances spatial imagination, the trigonometric method develops computational skills, and the analytical method perfectly demonstrates the application of algebra to geometry. Teachers are highly recommended to utilize such multi-method problems in their instructional process.

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