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YADROSIDA BESSEL FUNKSIYASI QATNASHGAN RIMAN-LIUVILL OPERATORINI O'Z ICHIGA OLUVCHI BIR ODDIY DIFFERENSIAL TENGLAMA UCHUN CHEGARAVIY MASALA

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Annotatsiya

Mazkur ishda yadrosida Bessel funksiyasi qatnashgan Riman-Liuvil operatorni o'z ichiga oluvchi bir oddiy differensial tenglama uchun chegaraviy masala yechimining mavjudligi va yagonalini isbotlangan.

КРАЕВАЯ ЗАДАЧА ПРОСТОГО ДИФФЕРЕНЦИАЛЬНОГО УРАВНЕНИЯ С ОПЕРАТОРОМ РИМАНА-ЛИУВИЛЛЯ, СОДЕРЖАЩИМ В ЯДРЕ ФУНКЦИЮ БЕССЕЛЯ

Аннотация

Доказано существование и единственность решения краевой задачи простого дифференциального уравнения, содержащего оператор Римана-Лиувилля с ядром, включающим функцию Бесселя

BOUNDARY VALUE PROBLEM FOR AN ORDINARY DIFFERENTIAL EQUATION CONTAINING THE GENERELIZED RIEMANN-LIOUVILLE FRACTIONAL OPERATOR WITH A KERNEL INVOLVING THE BESSEL FUNCTION

Annotation

The existence and uniqueness of the solution to the boundary value problem for an ordinary differential equation containing the Riemann-Liouville operator with a kernel involving the Bessel function have been proven.

Kalit so‘zlar

Riman-Liuvill integro-differensial operatori, chegaraviy masala, ketma-ket yaqinlashishlar usuli.

Ключевые слова

Интегро-дифференциальный оператор Римана-Лиувилля, краевая задача , метод последовательных приближений.

Key words

Riemann-Liouville integro-differential operator, boundary value problem, method of successive approximations.

1. Kirish. So‘nggi vaqtlarda tadqiqotchilar tomonidan kasr tartibli integral va differensial operatorlar va ular ishtirok etgan tenglamalarni o‘rganishga bo‘lgan qiziqish ortdi. Buni bir tomondan matematika fanining ichki ehtiyoji sifatida izohlansa, ikkinchi tomondan fan va texnikaning turli muammolarini matematik modellashtirishda shunday operatorlar ishtirok etgan tenglamalar hosil bo‘lishi bilan izohlash mumkin $[1],[2],[3]$. Bu yo‘nalishdagi tadqiqotlar turli yo‘nalishlarda rivojlanib bormoqda. Dastlabki tadqiqotlarda asosan Riman-Liuvill va Kaputo ma’nosidagi kasr tartibli integro-differensial operatorlar qaralgan bo‘lsa $[4]-[8]$, so‘nggi vaqtlarda ularning turli umumlashmalarini o‘rganishga bo‘lgan qiziqish ortdi $[9],[10],[11]$. Ushbu maqolada biz yadrosida Bessel funksiyasi qatnashgan umumlashgan Riman-Liuvill operatorini o‘z ichiga oluvchi oddiy differensial tenglama uchun chegaraviy masalasini bayon qilib, uning bir qiymatli yechilishini isbotlaymiz.

2. Masalaning qo‘yilishi. Ushbu tenglamani qaraylik:

$$\left(\frac{1+signx}{2}\right)D_{0x}^{\alpha,\gamma}y(x)+\left(\frac{1-signx}{2}\right)D_{x0}^{\alpha,\gamma}y(x)+\lambda signy(x)=f(x),\ x\in\left(-T,T\right),$$

(1)

bu yerda

$$signx=\begin{cases}1, & agar\quad x>0\quad bo'lsa\\ -1, & agar\quad x<0\quad bo'lsa\end{cases}$$

signx funksiya ko‘rinishini e’tiborga olib, (1) tenglamani quyidagi

$$f(x)=\begin{cases}D_{0x}^{\alpha,\gamma}y\left(x\right)+\lambda y\left(x\right), & 0<x<T\\ D_{x0}^{\alpha,\gamma}y\left(x\right)-\lambda y\left(x\right), & -T<x<0\end{cases} \qquad (2)$$

ko‘rinishda ham yozish mumkin, bu yerda $y(x)$ – noma’lum funksiya, $f(x)$ - berilgan funksiya, α,γ,λ,T lar esa berilgan haqiqiy sonlar bo‘lib, $3<\alpha<4$, $T>0$

$D_{ox}^{\alpha,\gamma}$ va $D_{x0}^{\alpha,\gamma}$ lar esa differensial operatorlar bo'lib, ushbu munosabatlar bilan aniqlanadi[11]:

$$D_{0x}^{\alpha,\gamma}y(x)=\left(\frac{d^2}{dx^2}+\gamma^2\right)^2 I_{0x}^{4-\alpha,\gamma}y(x), \qquad (3)$$

$$D_{x0}^{\alpha,\gamma}y(x)=\left(\frac{d^2}{dx^2}+\gamma^2\right)^2 I_{x0}^{4-\alpha,\gamma}y(x), \qquad (4)$$

$$I_{0x}^{\beta,\gamma}y(x)=\frac{1}{\Gamma(\beta)}\int_0^x (x-t)^{\beta-1}\bar{J}_{(\beta-1)/2}[\gamma(x-t)]y(t)dt, \qquad (5)$$

$$I_{x0}^{\beta,\gamma}y(x)=\frac{1}{\Gamma(\beta)}\int_x^0 (t-x)^{\beta-1}\bar{J}_{(\beta-1)/2}[\gamma(t-x)]y(t)dt\ , \qquad (6)$$

$\bar{J}_\nu(z)-$Bessel-Kliford funksiyasi bo'lib, quyidagi tenglik bilan aniqlanadi[11]:

$$\bar{J}_\nu(z)=\Gamma(\nu+1)(z/2)^{-\nu}J_\nu(z)=\sum_{k=0}^{\infty}\frac{(-1)^k(z/2)^{2k}}{k!(\nu+1)_k} \qquad (7)$$

$(z)_k$ $-$Poxgammer belgisi, $\Gamma(x)-$Eylerning gamma funksiyasi [12], $J_\nu(x)-$ birinchi tur Bessel funksiyasi [13].

$D_{0x}^{\alpha,\gamma}y(x)$ va $I_{0x}^{\beta,\gamma}y(x)$ operatorlar [11] ishda kiritilgan va ularning xossalari o'rganilgan bo'lib, ular mos holda Riman-Liuvill ma'nosidagi kasr tartibli differensial va integral operatorlarining umumlashmasidir.

(1) tenglama uchun quyidagi masalani o'rganamiz:

1-masala. (1) tenglamani va ushbu

$$\lim_{x\to a} I_{0x}^{4-\alpha,\gamma}y(x)=B_1, \qquad \lim_{x\to a}\frac{d}{dx}I_{0x}^{4-\alpha,\gamma}y(x)=B_3,$$
$$\lim_{x\to -a} I_{x0}^{4-\alpha,\gamma}y(x)=B_2, \qquad \lim_{x\to -a}\frac{d}{dx}I_{x0}^{4-\alpha,\gamma}y(x)=B_4 \qquad (8)$$

chegaraviy shartlarni hamda quyidagi

$$\lim_{x\to+0} I_{0x}^{4-\alpha,\gamma}y(x)=\lim_{x\to-0} I_{x0}^{4-\alpha,\gamma}y(x),$$
$$\lim_{x\to+0}\frac{d}{dx}I_{0x}^{4-\alpha,\gamma}y(x)=\lim_{x\to-0}\frac{d}{dx}I_{x0}^{4-\alpha,\gamma}y(x),$$
$$\lim_{x\to+0}\left(\frac{d^2}{dx^2}+\gamma^2\right)I_{0x}^{4-\alpha,\gamma}y(x)=\lim_{x\to-0}\left(\frac{d^2}{dx^2}+\gamma^2\right)I_{x0}^{4-\alpha,\gamma}y(x), \qquad (9)$$
$$\lim_{x\to+0}\frac{d}{dx}\left(\frac{d^2}{dx^2}+\gamma^2\right)I_{0x}^{4-\alpha,\gamma}y(x)=\lim_{x\to-0}\frac{d}{dx}\left(\frac{d^2}{dx^2}+\gamma^2\right)I_{x0}^{4-\alpha,\gamma}y(x)$$

ulash shartlarini qanoatlantiruvchi $y(x)$ funksiya topilsin, bu yerda B_1,B_2,B_3,B_4-berilgan haqiqiy sonlar.

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Masalaning tadqiqoti. Masala shartlariga asoslanib, quyidagi belgilashni kiritaylik:

$$\lim_{x\to+0} I_{0x}^{4-\alpha,\gamma} y(x) = \lim_{x\to-0} I_{x0}^{4-\alpha,\gamma} y(x) = A_1,$$

$$\lim_{x\to+0} \frac{d}{dx} I_{0x}^{4-\alpha,\gamma} y(x) = \lim_{x\to-0} \frac{d}{dx} I_{x0}^{4-\alpha,\gamma} y(x) = A_2,$$

$$\lim_{x\to+0} \left(\frac{d^2}{dx^2}+\gamma^2\right) I_{0x}^{4-\alpha,\gamma} y(x) = \lim_{x\to-0} \left(\frac{d^2}{dx^2}+\gamma^2\right) I_{x0}^{4-\alpha,\gamma} y(x) = A_3,$$

$$\lim_{x\to+0} \frac{d}{dx}\left(\frac{d^2}{dx^2}+\gamma^2\right) I_{0x}^{4-\alpha,\gamma} y(x) = \lim_{x\to-0} \frac{d}{dx}\left(\frac{d^2}{dx^2}+\gamma^2\right) I_{x0}^{4-\alpha,\gamma} y(x) = A_4$$

bu yerda A_1, A_2, A_3, A_4 – hozircha noma'lum sonlar.

Agar bu noma'lumlarni vaqtincha ma'lum deb faraz qilsak, u holdas faraz ostida $\{(1),(8)\}$ chegaraviy masalasining yechimini $(-T,0)$ va $(0,T)$ oraliqlar quyidagi ko'rinishlarda qidirishimiz mumkin bo'ladi:

$$y(x) = A_1(-x)^{\alpha-4} \mathrm{E}_{\alpha,\alpha-3,(\alpha-5)/2}\left[\lambda(-x)^{\alpha};\gamma(-x)\right] + A_2(-x)^{\alpha-3} \mathrm{E}_{\alpha,\alpha-2,(\alpha-3)/2}\left[\lambda(-x)^{\alpha};\gamma(-x)\right] +$$

$$+A_3(-x)^{\alpha-2} \mathrm{E}_{\alpha,\alpha-1,(\alpha-3)/2}[\lambda(-x)^{\alpha};\gamma(-x)] + A_4(-x)^{\alpha-1} \mathrm{E}_{\alpha,\alpha,(\alpha-1)/2}[\lambda(-x)^{\alpha};\gamma(-x)] +$$

$$+\int_{-x}^{0} (z-x)^{\alpha-1} \mathrm{E}_{\alpha,\alpha,(\alpha-1)/2}\left[\lambda(z-x)^{\alpha};\gamma(z-x)\right] f(z)dz, \quad x\in(-T,0), \qquad (10)$$

$$y(x) = A_1 x^{\alpha-4} \mathrm{E}_{\alpha,\alpha-3,(\alpha-5)/2}\left[-\lambda x^{\alpha};\gamma x\right] + A_2 x^{\alpha-3} \mathrm{E}_{\alpha,\alpha-2,(\alpha-3)/2}\left[-\lambda x^{\alpha};\gamma x\right] +$$

$$+A_3 x^{\alpha-2} \mathrm{E}_{\alpha,\alpha-1,(\alpha-3)/2}[-\lambda x^{\alpha};\gamma x] + A_4 x^{\alpha-1} \mathrm{E}_{\alpha,\alpha,(\alpha-1)/2}[-\lambda x^{\alpha};\gamma x] +$$

$$+\int_{0}^{x} (x-z)^{\alpha-1} \mathrm{E}_{\alpha,\alpha,(\alpha-1)/2}\left[-\lambda(x-z)^{\alpha};\gamma(x-z)\right] f(z)dz, \quad x\in(0,T), \qquad (11)$$

bu yerda

$$\mathrm{E}_{\alpha,\beta,\theta}[x;y] = \sum_{n=0}^{+\infty} \frac{x^n}{\Gamma(\alpha n+\beta)} \bar{J}_{\alpha n/2+\theta}(y). \qquad (12)$$

Ma'lumki, $\alpha>0, \beta>0$ bo'lganda (12) tenglik bilan aniqlangan qator $-\infty<x,y<\infty$ sohada absolyut va tekis yaqinlashuvchi bo'ladi [15].

Bundan tashqari (12) tenglik bilan aniqlangan funksiya uchun quyidagi

$$\mathrm{E}_{\alpha,\beta,\theta}[x;0] = E_{\alpha,\beta}(x), \mathrm{E}_{\alpha,\beta,\theta}[0;y] = \frac{1}{\Gamma(\beta)} \bar{J}_{\theta}(y), \mathrm{E}_{\alpha,\beta,\theta}[0,0] = \frac{1}{\Gamma(\beta)}$$

tengliklar va quyidagi hosila hisoblash formulalari o'rinli:

$$\frac{d}{dx}\mathrm{E}_{\alpha,1,(-1/2)}\left[-\lambda x^{\alpha};\gamma x\right] = -\lambda x^{\alpha-1}\mathrm{E}_{\alpha,\alpha,(\alpha-1)/2}\left[-\lambda x^{\alpha};\gamma x\right] - \gamma^2 x \mathrm{E}_{\alpha,2,1/2}\left[-\lambda x^{\alpha};\gamma x\right],$$

(13)

$$\frac{d}{dx}\left\{x^{\beta-1}\mathrm{E}_{\alpha,\beta,(\beta-1)/2}\left[-\lambda x^{\alpha};\gamma x\right]\right\}=x^{\beta-2}\mathrm{E}_{\alpha,\beta-1,(\beta-3)/2}\left[-\lambda x^{\alpha};\gamma x\right],\beta\neq 1.$$

(14)

(11) tenglik bilan aniqlanuvchi $y(x)$ funksiya (1) tenglamani $x\in(0,T)$ oraliqdagi yechimi ekanligining isboti [16] ishda keltirilgan. Endi (1) tenglamani $x\in(-T,0)$ boʻlgan hol uchun qaraymiz:

Shu maqsadda dastlab (1) tenglamaga $I_{x0}^{\alpha,\gamma}y(x)$ operatorni ta'sir ettiramiz, soʻngra ushbu

$$\left(I_{b-}^{\alpha,\lambda}D_{b-}^{\alpha,\lambda}f\right)(x)=f(x)-\frac{1}{\Gamma(\alpha)}\sum_{k=o}^{m}\left(\frac{d^2}{dx^2}+\lambda^2\right)^k\times$$

$$\times\left\{-(x-b)^{\alpha-1}\bar{J}_{(\alpha-1)/2}\left[\lambda(x-b)\right]\left[\frac{d}{dt}\left(\frac{d^2}{dt^2}+\lambda^2\right)^{m-k}I_{b-}^{2m+2-\alpha,\lambda}f\right]_{t=b}+\right.$$

$$\left.+(1-\alpha)(x-b)^{\alpha-2}\bar{J}_{(\alpha-3)/2}\left[\lambda(x-b)\right]\left[\left(\frac{d^2}{dx^2}+\lambda^2\right)^{m-k}I_{b-}^{2m+2-\alpha,\lambda}f(t)\right]_{t=0}\right\}$$

tenglikni e'tiborga olib [11], quyidagiga ega boʻlamiz:

$$I_{x0}^{\alpha,\gamma}D_{x0}^{\alpha,\gamma}y(x)=y(x)+\frac{x^{\alpha-1}}{\Gamma(\alpha)}\bar{J}_{(\alpha-1)/2}(\gamma x)\lim_{x\to-0}\frac{d}{dx}\left(\frac{d^2}{dx^2}+\gamma^2\right)I_{x0}^{4-\alpha,\gamma}y(x)+$$

$$+\frac{x^{\alpha-2}}{\Gamma(\alpha-1)}\bar{J}_{(\alpha-3)/2}(\gamma x)\lim_{x\to-0}\left(\frac{d^2}{dx^2}+\gamma^2\right)I_{x0}^{4-\alpha,\gamma}y(x)+$$

$$+\frac{1}{\Gamma(\alpha)}\left(\frac{d^2}{dx^2}+\gamma^2\right)x^{\alpha-1}\bar{J}_{(\alpha-1)/2}(\gamma x)\lim_{x\to-0}\frac{d}{dx}I_{x0}^{4-\alpha,\gamma}y(x)+$$

$$+\frac{1}{\Gamma(\alpha-1)}\left(\frac{d^2}{dx^2}+\gamma^2\right)x^{\alpha-2}\bar{J}_{(\alpha-1)/2}(\gamma x)\lim_{x\to-0}I_{x0}^{4-\alpha,\gamma}y(x).$$

(8) shartlarni e'tiborga olsak, oxirgi tenglik quyidagi koʻrinishni oladi:

$$y(x)-\lambda I_{x0}^{\alpha,\gamma}y(x)=I_{x0}^{\alpha,\gamma}f(x)-$$

$$-A_1\frac{1}{\Gamma(\alpha-1)}\left(\frac{d^2}{dx^2}+\gamma^2\right)x^{\alpha-2}\bar{J}_{(\alpha-3)/2}(\gamma x)-A_2\frac{1}{\Gamma(\alpha)}\left(\frac{d^2}{dx^2}+\gamma^2\right)x^{\alpha-1}\bar{J}_{(\alpha-1)/2}(\gamma x)-$$

$$-A_3\frac{x^{\alpha-2}}{\Gamma(\alpha-1)}\bar{J}_{(\alpha-3)/2}(\gamma x)-A_4\frac{x^{\alpha-1}}{\Gamma(\alpha)}\bar{J}_{(\alpha-1)/2}(\gamma x). \qquad (15)$$

Dastlab $\left(\frac{d^2}{dx^2}+\gamma^2\right)x^{\alpha-2}\overline{J}_{(\alpha-3)/2}(\gamma x)$ va $\left(\frac{d^2}{dx^2}+\gamma^2\right)x^{\alpha-1}\overline{J}_{(\alpha-1)/2}(\gamma x)$ ifodalarni soddalashtiraylik. $\overline{J}_{\frac{\alpha-3}{2}}(\gamma x)$ Bessel-Klifford funksiyasini uning (7) ifodasi bilan almashtirsak va kerakli hosilalarni hisoblasak, quyidagi natijaga ega boʻlamiz:

$$\left(\frac{d^2}{dx^2}+\gamma^2\right)x^{\alpha-2}\overline{J}_{(\alpha-3)/2}(\gamma x)=\frac{d^2}{dx^2}\sum_{k=0}^{\infty}\frac{(-1)^k(\gamma/2)^{2k}x^{2k+\alpha-2}}{k!((\alpha-1)/2)_k}+\gamma^2\sum_{k=0}^{\infty}\frac{(-1)^k(\gamma/2)^{2k}x^{2k+\alpha-2}}{k!((\alpha-1)/2)_k}=$$

$$=\sum_{k=0}^{\infty}\frac{(-1)^k(\gamma/2)^{2k}(2k+\alpha-2)(2k+\alpha-3)x^{2k+\alpha-4}}{k!((\alpha-1)/2)_k}+$$

$$+\gamma^2\sum_{k=0}^{\infty}\frac{(-1)^k(\gamma/2)^{2k}x^{2k+\alpha-2}}{k!((\alpha-1)/2)_k}. \qquad (16)$$

(16) ning oʻng tomonidagi yigʻindining birinchi hadi uchun quyida

$$\Gamma(a+n)=(a)_n\Gamma(a),\ (a)_{2n}=2^{2n}\left(\frac{a}{2}\right)_n\left(\frac{a+1}{2}\right)_n \qquad (17)$$

formulani ketma-ket qoʻllasak, quyidagi tenglik hosil boʻladi:

$$\left(\frac{d^2}{dx^2}+\gamma^2\right)x^{\alpha-2}\overline{J}_{(\alpha-3)/2}(\gamma x)=$$

$$=\sum_{k=0}^{\infty}\frac{(-1)^k(\gamma/2)^{2k}(2k+\alpha-2)(2k+\alpha-3)\Gamma((\alpha-3)/2)((\alpha-3)/2)x^{2k+\alpha-4}}{k!((2k+\alpha-3)/2)\Gamma((\alpha-3)/2+k)}+$$

$$+\gamma^2\sum_{k=0}^{\infty}\frac{(-1)^k(\gamma/2)^{2k}x^{2k+\alpha-2}}{k!((\alpha-1)/2)_k}.$$

Bundan esa baʼzi hisoblashlardan soʻng, ushbu

$$A_1\frac{1}{\Gamma(\alpha-1)}\left(\frac{d^2}{dx^2}+\gamma^2\right)x^{\alpha-2}\overline{J}_{(\alpha-3)/2}(\gamma x)=A_1\frac{x^{\alpha-4}}{\Gamma(\alpha-3)}\overline{J}_{(\alpha-5)/2}(\gamma x)$$

natijaga ega boʻlamiz.

$\left(\frac{d^2}{dx^2}+\gamma^2\right)x^{\alpha-1}\overline{J}_{(\alpha-1)/2}(\gamma x)$ funksiya uchun ifodani ham yuqoridagi kabi aniqlash mumkin. Topilgan natijalarni (15) tenglikka qoʻyib, quyidagi ikkinchi tur Volterra integral tenglamasiga ega boʻlamiz:

$$y(x)-\lambda I_{0x}^{\alpha,\gamma}y(x)=$$

$$=I_{0x}^{\alpha,\gamma} f(x)-A_1\frac{x^{\alpha-4}}{\Gamma(\alpha-3)}\overline{J}_{(\alpha-5)/2}(\gamma x)-A_2\frac{x^{\alpha-3}}{\Gamma(\alpha-2)}\overline{J}_{(\alpha-3)/2}(\gamma x)-$$

$$-A_3\frac{x^{\alpha-2}}{\Gamma(\alpha-1)}\overline{J}_{(\alpha-3)/2}(\gamma x)-A_4\frac{x^{\alpha-1}}{\Gamma(\alpha)}\overline{J}_{(\alpha-1)/2}(\gamma x). \qquad (18)$$

(18) integral tenglama yechimini ketma-ket yaqinlashishlar usuli bilan topamiz [14]. Nolinchi yaqinlashishni ushbu

$$y_0(x)=I_{0x}^{\alpha,\gamma} f(x)-A_1\frac{x^{\alpha-4}}{\Gamma(\alpha-3)}\overline{J}_{(\alpha-5)/2}(\gamma x)-A_2\frac{x^{\alpha-3}}{\Gamma(\alpha-2)}\overline{J}_{(\alpha-3)/2}(\gamma x)-$$

$$-A_3\frac{x^{\alpha-2}}{\Gamma(\alpha-1)}\overline{J}_{(\alpha-3)/2}(\gamma x)-A_4\frac{x^{\alpha-1}}{\Gamma(\alpha)}\overline{J}_{(\alpha-1)/2}(\gamma x)$$

tenglik bilan, qolgan yaqinlashishlarni esa

$$y_m(x)=y_0(x)+\lambda I_{0x}^{\alpha,\gamma} y_{m-1}(x),\ m\in N$$

munosabat bilan aniqlaymiz.

$I_{ax}^{\alpha,\gamma}I_{ax}^{\beta,\gamma}\varphi(x)=I_{ax}^{\beta,\gamma}I_{ax}^{\alpha,\gamma}\varphi(x)=I_{ax}^{\alpha+\beta,\gamma}\varphi(x)$ formuladan foydalanib, $y_m(x)$ ni quyidagi ko'rinishini yozib olamiz [14]:

$$y_m(x)=y_0(x)+\lambda I_{0x}^{\alpha,\gamma} y_0(x)+\lambda^2 I_{ox}^{2\alpha,\gamma} y_0(x)+\lambda^3 I_{0x}^{3\alpha,\gamma} y(x)+\ldots+\lambda^m I_{0x}^{\alpha m,\gamma} y_0(x). \quad (19)$$

(19) tenglikka $y_0(x)$ ning ifodasini qo'yib, $m\to\infty$ da limitga o'tsak,

$$y(x)=\frac{A_1}{\Gamma(\alpha-3)}\sum_{n=0}^{\infty}(\lambda)^n I_{x0}^{\alpha n,\gamma}\{x^{\alpha-4}\overline{J}_{(\alpha-5)/2}(\gamma x)\}-$$

$$-\frac{A_2}{\Gamma(\alpha-2)}\sum_{n=0}^{\infty}(\lambda)^n I_{x0}^{\alpha n,\gamma}\{x^{\alpha-3}\overline{J}_{(\alpha-3)/2}(\gamma x)\}-$$

$$-\frac{A_3}{\Gamma(\alpha-1)}\sum_{n=0}^{\infty}(\lambda)^n I_{x0}^{\alpha n,\gamma}\{x^{\alpha-2}\overline{J}_{(\alpha-3)/2}(\gamma x)\}-$$

$$-\frac{A_4}{\Gamma(\alpha)}\sum_{n=0}^{\infty}(\lambda)^n I_{x0}^{\alpha n,\gamma}\{x^{\alpha-1}\overline{J}_{(\alpha-1)/2}(\gamma x)\}+\sum_{n=0}^{\infty}(\lambda)^n I_{x0}^{\alpha n+\alpha,\gamma} f(x) \qquad (20)$$

tenglik kelib chiqadi.

(20) tenglikni soddalashtirish maqsadida $I_{x0}^{\alpha n,\gamma}\{x^{\alpha-4}\overline{J}_{(\alpha-5)/2}(\gamma x)\}$, $I_{x0}^{\alpha n,\gamma}\{x^{\alpha-3}\overline{J}_{(\alpha-3)/2}(\gamma x)\}$, $I_{x0}^{\alpha n,\gamma}\{x^{\alpha-2}\overline{J}_{(\alpha-3)/2}(\gamma x)\}$, $I_{x0}^{\alpha n,\gamma}\{x^{\alpha-1}\overline{J}_{(\alpha-1)/2}(\gamma x)\}$ va $I_{x0}^{\alpha n+\alpha,\gamma} f(x)$ integrallarni qaraymiz.

$I_{x0}^{\alpha n,\gamma}\{x^{\alpha-4}\overline{J}_{(\alpha-5)/2}(\gamma x)\}$ uchun (6) ga ko'ra, ushbu tenglik o'rinli:

$$I_{x0}^{\alpha n,\gamma}\{x^{\alpha-4}\overline{J}_{(\alpha-5)/2}(\gamma x)\}=$$

$$=\frac{1}{\Gamma(\alpha n)}\int_x^0 z^{\alpha-4}(z-x)^{\alpha n-1}\overline{J}_{(\alpha n-1)/2}[\gamma(z-x)]\overline{J}_{(\alpha-5)/2}(\gamma z)dz. \qquad (21)$$

(21) dagi $\overline{J}_\nu(x)$ funksiyalarni ularning (7) ifodasi bilan almashtirsak, quyidagiga ega boʻlamiz:

$$\overline{J}_{(\alpha n-1)/2}[\gamma(z-x)]\overline{J}_{(\alpha-5)/2}[\gamma(z)]=$$

$$=\sum_{m=0}^{\infty}\frac{(-1)^m(\gamma/2)^{2m}(z-x)^{2m}}{m!((\alpha n+1)/2)_m}\sum_{m=0}^{\infty}\frac{(-1)^k(\gamma/2)^{2k}z^{2k}}{k!((\alpha-3)/2)_k}.$$

Bu yerdan yaqinlashuvchi qatorlarni koʻpaytirishning Koshi qoidasini qoʻllab, quyidagi natijaga kelamiz:

$$\overline{J}_{(\alpha n-1)/2}[\gamma(z-x)]\overline{J}_{(\alpha-5)/2}[\gamma(z)]=$$

$$=\sum_{m=0}^{\infty}\sum_{k=0}^{m}\frac{(-1)^k(\gamma/2)^{2k}(z-x)^{2k}}{k!((\alpha n+1)/2)_k}\frac{(-1)^{m-k}(\gamma/2)^{2m-2k}(z)^{2m-2k}}{(m-k)!((\alpha-3)/2)_{m-k}}=$$

$$=\sum_{m=0}^{\infty}(-1)^m(\gamma/2)^{2m}\sum_{k=0}^{m}\frac{z^{2m-2k}(z-x)^{2k}}{k!(m-k)!((\alpha n+1)/2)_k((\alpha-3)/2)_{m-k}}.$$

Olingan natijani (21) tenglikka qoʻyib, integral va yigʻindining tartibini oʻzgartirsak,

$$I_{x0}^{\alpha n,\gamma}\{x^{\alpha-4}\overline{J}_{(\alpha-5)/2}(\gamma x)\}=\sum_{m=0}^{\infty}\frac{(-1)^m(\gamma/2)^{2m}}{\Gamma(\alpha n)}\times$$

$$\times\sum_{k=0}^{m}\frac{1}{k!(m-k)!((\alpha n+1)/2)_k((\alpha-1)/2)_{m-k}}\int_x^0 z^{2m+\alpha-2k-4}(z-x)^{\alpha n+2k-1}dz. \qquad (22)$$

Ichki integralda integrallash oʻzgaruvchisini $z=-xs+x$ formula boʻyicha almashtirish bajarib, baʼzi hisoblashlardan soʻng, quyidagi natijaga ega boʻlamiz:

$$\int_x^0 z^{2m+\alpha-2k-4}(z-x)^{\alpha n+2k-1}dz=$$

$$=(-1)^{\alpha n}x^{\alpha n+2m+\alpha-4}\Gamma(2m-2k+\alpha-3)\Gamma(\alpha n+2k)/\Gamma(\alpha n+2m+\alpha-3).$$

Hosil boʻlgan natijani (22) ga qoʻyib va (17) formulalarni ketma-ket qoʻllasak, ushbu tenglik hosil boʻladi:

$$I_{x0}^{\alpha n,\gamma}\{x^{\alpha-4}\overline{J}_{(\alpha-5)/2}(\gamma x)\}=$$

$$=\Gamma(\alpha-3)\sum_{m=0}^{\infty}\frac{(-1)^{m+\alpha n}(\gamma/2)^{2m}x^{\alpha n+2m+\alpha-4}}{\Gamma(\alpha n+2m+\alpha-3)}\sum_{k=0}^{m}\frac{(\alpha n)_{2k}(\alpha-3)_{2m-2k}}{k!(m-k)!((\alpha n+1)/2)_k((\alpha-1)/2)_{m-k}}=$$

$$=\Gamma(\alpha-3)\sum_{m=0}^{\infty}\frac{(-1)^{m+\alpha n}(\gamma/2)^{2m}2^{2m}x^{\alpha n+2m+\alpha-4}}{\Gamma(\alpha n+2m+\alpha-3)}\sum_{k=0}^{m}\frac{(\alpha n/2)_k((\alpha-2)/2)_{m-k}}{k!(m-k)!}.$$

Ushbu

$$\sum_{k=0}^{m}\frac{(\delta)_k(\gamma)_{m-k}}{k!(m-k)!}=\frac{(\delta+\gamma)_m}{m!} \qquad (23)$$

formulani e'tiborga olsak, oxirgi tenglik ushbu ko'rinishni oladi:

$$I_{x0}^{\alpha n,\gamma}\{x^{\alpha-4}\overline{J}_{(\alpha-5)/2}(\gamma x)\}=\Gamma(\alpha-1)\sum_{m=0}^{\infty}\frac{(-1)^{m+\alpha n}\gamma^{2m}((\alpha n+\alpha-2)/2)_m}{m!\Gamma(\alpha n+2m+\alpha-3)}.$$

Bu yerdan $\Gamma(\alpha n+2m+\alpha-3)$ uchun (17) formulalarni qo'llab, oxirgi ifodani ushbu ko'rinishga keltiramiz:

$$I_{x0}^{\alpha n,\gamma}\{x^{\alpha-4}\overline{J}_{(\alpha-5)/2}(\gamma x)\}=\frac{(-1)^{\alpha n}\Gamma(\alpha-3)}{\Gamma(\alpha n+\alpha-3)}\sum_{m=0}^{\infty}\frac{(-1)^m(\gamma/2)^{2m}x^{\alpha n+2m+\alpha-4}}{m!((\alpha n+\alpha-3)/2)_m}.$$

Oxirgidan (7) tenglikka asosan ushbu

$$I_{x0}^{\alpha n,\gamma}\{x^{\alpha-4}\overline{J}_{(\alpha-5)/2}(\gamma x)\}=(-1)^{\alpha n}\frac{x^{\alpha n+\alpha-4}\Gamma(\alpha-3)}{\Gamma(\alpha n+\alpha-3)}\overline{J}_{((\alpha n+\alpha-5)/2)}(\gamma x) \qquad (24)$$

natijaga ega bo'lamiz.

Yuqoridagiga o'xshash amallar bajarib, ushbu

$$I_{0x}^{\alpha n,\gamma}\{x^{\alpha-3}\overline{J}_{(\alpha-3)/2}(\gamma x)\}=(-1)^{\alpha n}\frac{x^{\alpha n+\alpha-3}\Gamma(\alpha-2)}{\Gamma(\alpha n+\alpha-2)}\overline{J}_{((\alpha n+\alpha-3)/2)}(\gamma x), \qquad (25)$$

$$I_{0x}^{\alpha n,\gamma}\{x^{\alpha-2}\overline{J}_{(\alpha-3)/2}(\gamma x)\}=(-1)^{\alpha n}\frac{x^{\alpha n+\alpha-2}\Gamma(\alpha-1)}{\Gamma(\alpha n+\alpha-1)}\overline{J}_{((\alpha n+\alpha-3)/2)}(\gamma x), \qquad (26)$$

$$I_{0x}^{\alpha n,\gamma}\{x^{\alpha-1}\overline{J}_{(\alpha-1)/2}(\gamma x)\}=(-1)^{\alpha n}\frac{x^{\alpha n+\alpha-1}\Gamma(\alpha)}{\Gamma(\alpha n+\alpha)}\overline{J}_{((\alpha n+\alpha-1)/2)}(\gamma x) \qquad (27)$$

tengliklar o'rinli ekanligini ko'rsatish mumkin.

(24), (25), (26) va (27) larni (20) tenglikka qo'yib va (6) operatorning yoyilmasini e'tiborga olib, (10) formulaga ega bo'lamiz.

Endi (10) formula bilan aniqlangan $y(x)$ funksiyani (1) tenglamani va (9) shartni qanoatlantirishini ko'rsatamiz. Shu maqsadda uni ushbu

$$y(x)=y_1(x)+y_2(x)+y_3(x)+y_4(x)+y_5(x)$$

ko'rinishida yozib olamiz, bu yerda

$$y_1(x)=(-1)^{\alpha+1}A_1x^{\alpha-4}\mathrm{E}_{\alpha,\alpha-3,(\alpha-5)/2}\left[-\lambda x^{\alpha};\gamma x\right],$$

$$y_2(x)=(-1)^{\alpha+1}A_2x^{\alpha-3}\mathrm{E}_{\alpha,\alpha-2,(\alpha-3)/2}\left[-\lambda x^{\alpha};\gamma x\right],$$

$$y_3(x)=(-1)^{\alpha+1}A_3x^{\alpha-2}\mathrm{E}_{\alpha,\alpha-1,(\alpha-3)/2}[-\lambda x^{\alpha};\gamma x],$$

$$y_4(x)=(-1)^{\alpha+1}A_4x^{\alpha-1}\mathrm{E}_{\alpha,\alpha,(\alpha-1)/2}[-\lambda x^{\alpha};\gamma x],$$

$$y_5(x)=\int_x^0(z-x)^{\alpha-1}\mathrm{E}_{\alpha,\alpha,(\alpha-1)/2}\left[\lambda(z-x)^{\alpha};\gamma(z-x)\right]f(z)dz.$$

(4) formulani e'tiborga olib, dastlab $I_{x0}^{4-\alpha,\gamma}y_1(x)$ ni hisoblaymiz:

$$I_{x0}^{4-\alpha,\gamma}y_1(x)=\frac{1}{\Gamma(4-\alpha)}\int_x^0(z-x)^{3-\alpha}\bar{J}_{((3-\alpha)/2)}\left[\gamma(z-x)\right]y_1(z)dz=$$

$$=\frac{A_1}{\Gamma(4-\alpha)}\int_x^0(z-x)^{3-\alpha}z^{\alpha-4}\bar{J}_{((3-\alpha)/2)}\left[\gamma(z-x)\right]\mathrm{E}_{\alpha,\alpha-3,((\alpha-5)/2)}\left[-\lambda z^{\alpha};\gamma z\right]dz=$$

$$=A_1\sum_{n=0}^{+\infty}\frac{(-\lambda)^n}{\Gamma(\alpha n+\alpha-3)}\frac{1}{\Gamma(4-\alpha)}\int_x^0(z-x)^{3-\alpha}z^{\alpha n+\alpha-4}\bar{J}_{(3-\alpha)/2}\left[\gamma(z-x)\right]\bar{J}_{((\alpha n+\alpha-5)/2))}(\gamma z)dz.$$

Quyidagi belgilashni kiritaylik:

$$G(\alpha,n,\gamma;x)=$$

$$=\frac{1}{\Gamma(\alpha n+\alpha-3)\Gamma(4-\alpha)}\int_x^0(z-x)^{3-\alpha}z^{\alpha n+\alpha-4}\bar{J}_{(3-\alpha)/2}\left[\gamma(z-x)\right]\bar{J}_{(\alpha n+\alpha-5)/2}(\gamma z)dz. \quad (28)$$

U holda oxirgi tenglik quyidagicha yoziladi:

$$I_{x0}^{4-\alpha,\gamma}y_1(x)=(-1)^{\alpha+1}A_1\sum_{n=0}^{\infty}(-\lambda)^nG(\alpha,n,\gamma;x). \quad (29)$$

Endi $G(\alpha,n,\gamma;x)$ funksiyani soddalashtiraylik. Shu maqsadda $\bar{J}_{\nu}(x)$ funksiyani uning (7) ifodasi bilan almashtirib, qatorlarni ko'paytirishning Koshi qoidasini qo'llab, quyidagini hosil qilamiz:

$$\bar{J}_{(3-\alpha)/2}\left[\gamma(z-x)\right]\bar{J}_{(\alpha n+\alpha-5)/2}(\gamma z)=$$

$$=\sum_{m=0}^{\infty}(-1)^m(\gamma/2)^{2m}\sum_{k=0}^{m}\frac{z^{2k}(z-x)^{2m-2k}}{k!(m-k)!((5-\alpha)/2)_{m-k}((\alpha n+\alpha-3)/2)_k}.$$

Oxirgini e'tiborga olib, (29) dan quyidagini hosil qilamiz:

$$G(\alpha,n,\gamma;x)=$$

$$=\sum_{m=0}^{\infty}\frac{(-1)^m(\gamma/2)^{2m}}{\Gamma(\alpha n+\alpha-3)\Gamma(4-\alpha)}\sum_{k=0}^{m}\frac{G_1(\alpha,n,m,k;x)}{k!(m-k)!((5-\alpha)/2)_{m-k}((\alpha n+\alpha-3)/2)_k},$$

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bu yerda

$$G_1(\alpha,n,m,k;x)=\int_x^0 z^{\alpha n+\alpha+2k-4}(z-x)^{3+2m-2k-\alpha}\,dz.$$

Ko'rsatish qiyin emaski, $G_1(\alpha,n,m,k;x)$ funksiya uchun ushbu tenglik o'rinli:

$$G_1(\alpha,n,m,k;x)=(-1)^{3-\alpha}x^{\alpha n+2m}\frac{\Gamma(2m-2k+4-\alpha)\Gamma(\alpha n+\alpha+2k-3)}{\Gamma(\alpha n+2m+1)}.$$

Bu natijani (29) ga qo'yib, $\Gamma(\alpha n+\alpha+2k-3)$ va $\Gamma(2m-2k+4-\alpha)$ lar uchun (17) formulani ketma-ket qo'llasak, quyidagi natijaga ega bo'lamiz:

$$G(\alpha,n,\gamma;x)=$$

$$=\sum_{m=0}^{\infty}\frac{(-1)^{m+3-\alpha}(\gamma/2)^{2m}x^{2m+\alpha n}}{\Gamma(\alpha n+2m+1)}\sum_{k=0}^{m}\frac{(4-\alpha)_{2m-2k}(\alpha n+\alpha-3)_{2k}}{k!(m-k)!((\alpha n+\alpha-3)/2)_k((5-\alpha)/2)_{m-k}}=$$

$$=\sum_{m=0}^{\infty}\frac{(-1)^{m+3-\alpha}\gamma^{2m}x^{2m+\alpha n}}{\Gamma(\alpha n+2m+1)}\sum_{k=0}^{m}\frac{((4-\alpha)/2)_{m-k}((\alpha n+\alpha-2)/2)_k}{k!(m-k)!}.$$

Agar (23) tenglikni hisobga olsak, $H(\alpha,n,\gamma;x)$ funksiya quyidagi ko'rinishni oladi:

$$G(\alpha,n,\gamma;x)=\sum_{m=0}^{\infty}\frac{(-1)^{m+3-\alpha}\gamma^{2m}((\alpha n+\alpha)/2)_k x^{2m+\alpha n}}{m!\Gamma(\alpha n+2m+1)}. \qquad (30)$$

$G(\alpha,n,\gamma;x)$ funksiyaning (30) ifodasini (29) qo'yib, ushbu

$$I_{x0}^{4-\alpha,\gamma}y_1(x)=A_1\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-\lambda)^n(-1)^m\gamma^{2m}((\alpha n+\alpha)/2)_m x^{2m+\alpha n}}{m!\Gamma(\alpha n+2m+1)}$$

tenglikni hosil qilamiz.

$\Gamma(\alpha n+2m+1)$ uchun (17) formulalarni ketma-ket qo'llab, va (10) ni e'tiborga olgan holda, ushbu

$$I_{x0}^{4-\alpha,\gamma}y_1(x)=A_1\mathrm{E}_{\alpha,1,-1/2}\left[-\lambda x^{\alpha};\gamma x\right] \qquad (31)$$

natijaga ega bo'lamiz.

Yuqoridagi kabi ushbu

$$I_{x0}^{4-\alpha,\gamma}y_2(x)=A_2x\mathrm{E}_{\alpha,2,1/2}\left[-\lambda x^{\alpha};\gamma x\right], \qquad (32)$$

$$I_{x0}^{4-\alpha,\gamma}y_3(x)=A_3x^2\mathrm{E}_{\alpha,3,1/2}\left[-\lambda x^{\alpha};\gamma x\right], \qquad (33)$$

$$I_{x0}^{4-\alpha,\gamma}y_4(x)=A_4x^3\mathrm{E}_{\alpha,4,3/2}\left[-\lambda x^{\alpha};\gamma x\right], \qquad (34)$$

$$I_{x0}^{4-\alpha,\gamma}y_5(x)=\int_x^0(z-x)^3\mathrm{E}_{\alpha,4,3/2}\left[\lambda(z-x)^{\alpha};\gamma(z-x)\right]f(z)\,dz \qquad (35)$$

tengliklarning o'rinli ekanligi ham ko'rsatish mumkin.

(31), (32), (33), (34), (35) tengliklarni e'tiborga olib, $I_{x0}^{4-\alpha,\gamma}y(x)$ funksiya uchun ushbu ifodani hosil qilamiz:

$$I_{x0}^{4-\alpha,\gamma}y(x)=A_1\mathrm{E}_{\alpha,1,-1/2}\left[-\lambda x^{\alpha};\gamma x\right]+A_2x\mathrm{E}_{\alpha,2,1/2}\left[-\lambda x^{\alpha};\gamma x\right]+A_3x^2\mathrm{E}_{\alpha,3,1/2}\left[-\lambda x^{\alpha};\gamma x\right]+$$
$$+A_4x^3\mathrm{E}_{\alpha,4,3/2}\left[-\lambda x^{\alpha};\gamma x\right]+\int_x^0(z-x)^3\mathrm{E}_{\alpha,4,3/2}\left[\lambda(z-x)^{\alpha};\gamma(z-x)\right]f(z)dz. \qquad (36)$$

$D_{x0}^{\alpha,\gamma}y(x)=\left(\frac{d^2}{dx^2}+\gamma^2\right)^2 I_{x0}^{4-\alpha,\gamma}y(x)$ ekanligini va (13), (14) hamda $\mathrm{E}_{\alpha,\beta,\theta}[0;0]=\frac{1}{\Gamma(\beta)}$ tengliklarni e'tiborga olib, ushbu

$$\left(\frac{d^2}{dx^2}+\gamma^2\right)^2 I_{x0}^{4-\alpha,\gamma}y(x)=$$
$$A_1\left(\frac{d^2}{dx^2}+\gamma^2\right)^2\mathrm{E}_{\alpha,1,-1/2}\left[-\lambda x^{\alpha};\gamma x\right]+A_2\left(\frac{d^2}{dx^2}+\gamma^2\right)^2x\mathrm{E}_{\alpha,2,1/2}\left[-\lambda x^{\alpha};\gamma x\right]+$$
$$+A_3\left(\frac{d^2}{dx^2}+\gamma^2\right)^2x^2\mathrm{E}_{\alpha,3,1/2}\left[-\lambda x^{\alpha};\gamma x\right]+A_4\left(\frac{d^2}{dx^2}+\gamma^2\right)^2x^3\mathrm{E}_{\alpha,4,3/2}\left[-\lambda x^{\alpha};\gamma x\right]+$$
$$+\left(\frac{d^2}{dx^2}+\gamma^2\right)^2\int_x^0(z-x)^3\mathrm{E}_{\alpha,4,3/2}\left[\lambda(z-x)^{\alpha};\gamma(z-x)\right]f(z)dz$$

tenglikni hosil qilamiz. Oxirgi tenglikka (13) va (14) formulalarni ketma-ket qo'llab, $\left(\frac{d^2}{dx^2}+\gamma^2\right)\mathrm{E}_{\alpha,1,-1/2}\left[-\lambda x^{\alpha};\gamma x\right]$ni quyidagi ko'rinishda yozishimiz mumkin:

$$\left(\frac{d^2}{dx^2}+\gamma^2\right)\mathrm{E}_{\alpha,1,-1/2}\left[-\lambda x^{\alpha};\gamma x\right]=-\lambda x^{\alpha-2}\mathrm{E}_{\alpha,\alpha-1,(\alpha-3)/2}\left[-\lambda x^{\alpha};\gamma x\right].$$

Endi (12) va (7) tengliklarga asosan $\left(\frac{d^2}{dx^2}+\gamma^2\right)x^{\alpha-2}\mathrm{E}_{\alpha,\alpha-1,(\alpha-3)/2}\left[-\lambda x^{\alpha};\gamma x\right]$ funksiyani quyidagi ko'rinishda yozib olamiz:

$$\left(\frac{d^2}{dx^2}+\gamma^2\right)x^{\alpha-2}\mathrm{E}_{\alpha,\alpha-1,(\alpha-3)/2}\left[-\lambda x^{\alpha};\gamma x\right]=$$
$$=\left(\frac{d^2}{dx^2}+\gamma^2\right)\sum_{n=0}^{\infty}\frac{(-\lambda)^n}{\Gamma(\alpha n+\alpha-1)}\sum_{m=0}^{\infty}\frac{(-1)^m(\gamma/2)^{2m}x^{2m+\alpha n+\alpha-2}}{m!\left((\alpha n+\alpha-1)/2\right)_m}.$$

Oxirgi tenglikning o'ng tarafini hadma-had ko'paytirib, kerakli hosilani hisoblaymiz va $\left((\alpha n+\alpha-1)/2\right)_m$ ifoda uchun (17) formulani qo'llab, uni quyidagi ko'rinishga keltiramiz:

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$$\left(\frac{d^2}{dx^2}+\gamma^2\right)x^{\alpha-2}\mathrm{E}_{\alpha,\alpha-1,(\alpha-3)/2}\left[-\lambda x^{\alpha};\gamma x\right]=$$

$$=x^{\alpha-4}\sum_{n=0}^{\infty}\frac{(-\lambda)^n x^{\alpha n}}{\Gamma(\alpha n+\alpha-3)}\sum_{m=0}^{\infty}\frac{(-1)^m(\gamma/2)^{2m}x^{2m}}{m!\left((\alpha n+\alpha-3)/2\right)_m}.$$

U holda (12) tenglikka asosan, ushbu

$$\left(\frac{d^2}{dx^2}+\gamma^2\right)^2 I_{x0}^{4-\alpha,\gamma}y_1(x)=\lambda A_1x^{\alpha-4}\mathrm{E}_{\alpha,\alpha-3,(\alpha-5)/2}\left[-\lambda x^{\alpha};\gamma x\right] \qquad (37)$$

natija kelib chiqadi.

Yuqoridagi kabi hisoblashlar bilan $\left(\frac{d^2}{dx^2}+\gamma^2\right)^2 I_{x0}^{4-\alpha,\gamma}y(x)$ tenglikning boshqa hadlarini ham soddalashtirish mumkin. Shunday qilib, (36) va (37) tengliklardan shunday natijaga kelamiz:

$$D_{x0}^{\alpha,\gamma}y(x)=\left(\frac{d^2}{dx^2}+\gamma^2\right)^2 I_{x0}^{4-\alpha,\gamma}y(x)=+\lambda A_1x^{\alpha-4}\mathrm{E}_{\alpha,\alpha-3,(\alpha-5)/2}\left[-\lambda x^{\alpha};\gamma x\right]+$$
$$+\lambda A_2x^{\alpha-3}\mathrm{E}_{\alpha,\alpha-2,(\alpha-3)/2}\left[-\lambda x^{\alpha};\gamma x\right]+$$
$$+\lambda A_3x^{\alpha-2}\mathrm{E}_{\alpha,\alpha-1,(\alpha-3)/2}[-\lambda x^{\alpha};\gamma x]+\lambda A_4x^{\alpha-1}\mathrm{E}_{\alpha,\alpha,(\alpha-1)/2}[-\lambda x^{\alpha};\gamma x]-$$
$$-f(x)+\lambda\int_x^0(z-x)^{\alpha-1}\mathrm{E}_{\alpha,\alpha,(\alpha-1)/2}\left[\lambda(z-x)^{\alpha};\gamma(z-x)\right]f(z)dz. \qquad (38)$$

Topilgan (38) hamda (10) tengliklarni taqqoslab, (10) formula bilan aniqlangan $y(x)$ funksiya (1) tenglamani $(-T;0)$ oraliqda qanoatlantiradi degan xulosaga kelamiz.

Endi (10), (11) yechimlarni (8) shartga bo'ysundiramiz, natijada B_1,B_2,B_3,B_4 larga nisbatan ushbu

$$B_1=A_1\mathrm{E}_{\alpha,1,-1/2}\left[-\lambda a^{\alpha};\gamma a\right]+A_2a\mathrm{E}_{\alpha,2,1/2}\left[-\lambda a^{\alpha};\gamma a\right]+A_3a^2\mathrm{E}_{\alpha,3,1/2}\left[-\lambda a^{\alpha};\gamma a\right]+$$
$$+A_4a^3\mathrm{E}_{\alpha,4,3/2}\left[-\lambda a^{\alpha};\gamma a\right]+\int_0^a(a-z)^3\mathrm{E}_{\alpha,4,3/2}\left[-\lambda(a-z)^{\alpha};\gamma(a-z)\right],$$

$$B_2=A_1\mathrm{E}_{\alpha,1,-1/2}\left[\lambda(-a)^{\alpha};-\gamma a\right]+A_2(-a)\mathrm{E}_{\alpha,2,1/2}\left[\lambda(-a)^{\alpha};-\gamma a\right]+A_3a^2\mathrm{E}_{\alpha,3,1/2}\left[\lambda(-a)^{\alpha};-\gamma a\right]+$$
$$+A_4(-a)^3\mathrm{E}_{\alpha,4,3/2}\left[\lambda(-a)^{\alpha};-\gamma a\right]+\int_{-a}^0(z-a)^3\mathrm{E}_{\alpha,4,3/2}\left[\lambda(z-a)^{\alpha};\gamma(z-a)\right],$$

$$B_3=-A_1a^{\alpha-1}\lambda\mathrm{E}_{\alpha,\alpha,(\alpha-1)/2}\left[-\lambda a^{\alpha};\gamma a\right]-\gamma^2A_1\mathrm{E}_{\alpha,2,1/2}\left[-\lambda a^{\alpha};\gamma a\right]+A_2\mathrm{E}_{\alpha,1,-1/2}\left[-\lambda a^{\alpha};\gamma a\right]+$$

$$+A_3 a\sum_{n=0}^{\infty}\frac{\left(-\lambda a^{\alpha}\right)^n}{\Gamma(\alpha n+3)}\sum_{m=0}^{\infty}\frac{(-1)^m(\gamma/2)^{2m}a^{2m}(2m+\alpha n+2)}{m!(\alpha n+3/2)_m}+A_4 a^2\mathrm{E}_{\alpha,3,1/2}\left[-\lambda a^{\alpha};\gamma a\right]+$$

$$+\int_0^a (a-z)^2\,\mathrm{E}_{\alpha,3,1/2}\left[-\lambda(a-z)^{\alpha};\gamma(a-z)\right],$$

$$B_4=-A_1(-a)^{\alpha-1}\lambda\mathrm{E}_{\alpha,\alpha,(\alpha-1)/2}\left[\lambda(-a)^{\alpha};-\gamma a\right]-\gamma^2A_1\mathrm{E}_{\alpha,2,1/2}\left[\lambda(-a)^{\alpha};-\gamma a\right]+$$

$$+A_2\mathrm{E}_{\alpha,1,-1/2}\left[\lambda(-a)^{\alpha};-\gamma a\right]-A_3 a\sum_{n=0}^{\infty}\frac{\left(\lambda(-a)^{\alpha}\right)^n}{\Gamma(\alpha n+3)}\sum_{m=0}^{\infty}\frac{(-1)^m(\gamma/2)^{2m}a^{2m}(2m+\alpha n+2)}{m!(\alpha n+3/2)_m}+$$

$$+A_4 a^2\mathrm{E}_{\alpha,3,1/2}\left[\lambda(-a)^{\alpha};-\gamma a\right]+\int_{-1}^{0}(z-a)^2\,\mathrm{E}_{\alpha,3,1/2}\left[\lambda(z-a)^{\alpha};\gamma(z-a)\right]$$

tenglamalar sistemasini hosil qilamiz. Bu sistemaning yechimi mavjud va yagona bo'lishi uchun uning asosiy determinanti noldan farqli bo'lishi kerak. Uning asosiy determinantini Δ orqali belgilaylik.

Quyidagi teorema o'rinli:

1-teorema. *Agar* $\Delta\neq 0$ *bo'lib,* $f(x)=x^{-p}f_1(x)$, $f_1(x)\in C[-T,T]$, $0\le p<1$ *bo'lsa, u holda {(1),(8)} masala yagona yechimga ega bo'ladi.*

Isbot. $\Delta\neq 0$ bo'lsin. U holda yuqoridagi tenglamalar sitemasidan B_1,B_2,B_3,B_4 noma'lumlar bir qiymatli aniqlanadi. Ularning topilgan qiymatlarini (10), (11) yechimlarga qo'yib, {(1),(8)} masalaning yechimiga ega bo'lamiz.

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