

METHODOLOGY FOR SOLVING PROBLEMS RELATED TO THE THIN LENS FORMULA

<https://doi.org/10.5281/zenodo.18446480>

Abdulla Abduraimovich Uzoqov¹, Khudaybergan Madraimov Atabek o'g'li²,

Muxlisa Abdumajid qizi Abdumalikova³

¹*National Pedagogical University of Uzbekistan, Tashkent 100070, Uzbekistan,
Associate Professor, PhD,*

²*National Pedagogical University of Uzbekistan, Tashkent 100070, Uzbekistan, 2nd–
year Master's Student,*

³*National Pedagogical University of Uzbekistan, Tashkent 100070, Uzbekistan, 4th–
year Student.*

e-mail: uzoqov@gmail.com¹, xudayberganmadraimov8@gmail.com²

Abstract

This article is devoted to thin lenses and outlines methodologies for solving problems based on the formulas for the focal length and optical power of thin lenses. Solving these problems requires a thorough understanding of image formation by thin lenses.

Keywords

thin lens, focus of the lens, optical power, radius of curvature, refractive index.

Аннотация

Данная статья посвящена тонким линзам и в ней приводится методика решения задач, используя формулы нахождения фокусного расстояния и оптической силы тонких линз. Для решения этих задач нужно уметь получать изображения на тонких линзах.

Ключевые слова

тонкая линза, фокус линзы, оптическая сила, радиус кривизны, показатель преломления.

The training of mature and highly qualified teaching personnel for Uzbekistan republic is now regarded as one of the crucial issues. For this reason, raising the level and quality of knowledge among students in higher education institutions remains a highly relevant problem. When students completing higher education enter general secondary schools, they need to become strong experts in their respective disciplines. To achieve this, prospective physics teachers must obtain

thorough knowledge during their university years and engage in substantial self-development.

This article will prove beneficial for students pursuing higher education as well as for pupils in general secondary schools in their preparation for physics olympiads. It outlines methodological approaches to solving several advanced problems pertaining to the **optics** section of physics, specifically those involving **thin lens formulas** and grounded in the **law of refraction** (Snell's law). Lenses function based on the **law of refraction of light** and play a pivotal role in the **image formation** of objects. They find applications across diverse domains of human endeavor – from everyday devices (cameras, eyeglasses, etc.) to sophisticated scientific instruments (microscopes, binoculars, telescopes, etc.). The fundamental principle of a lens lies in altering the direction of propagation of light rays, thereby enabling the production of **magnified** or **minified images**. Additionally, lenses serve to correct optical aberrations and refractive errors of the human eye. Depending on the type of lens and the position of the object, the resulting picture can be real or virtual, enlarged or reduced, and upright or inverted.

The thin lens equation [1] is expressed as follows:

$$\frac{1}{F} = \frac{1}{a} + \frac{1}{b'} \quad (1)$$

in the equation above: a – represents the distance from the object to the optical center of the lens, b' – represents the distance from the image to the optical center of the lens, and F – represents the principal focal length of the lens.

The formula for a thin lens can also be written in terms of the radii of curvature of the spherical surfaces that bound it [1]:

$$\frac{1}{F} = \frac{n_L - n_m}{n_m} \left(\frac{1}{R_1} + \frac{1}{R_2} \right), \quad (2)$$

here, n_L – refractive index of the lens, n_m – refractive index of the medium, R_1 and R_2 – radii of curvature of the side surfaces of the lens.

The optical power of a thin lens is as follows [2]:

$$D = \frac{n_m}{F} = (n_L - n_m) \left(\frac{1}{R_1} + \frac{1}{R_2} \right). \quad (3)$$

If two thin lenses are separated by a distance l , their equivalent optical power is determined as follows

$$D = D_1 + D_2 - D_1 D_2 l. \quad (4)$$

where D_1 and D_2 are the optical powers of the lenses, and l is the distance between them. If the lenses are placed in contact with each other, then,

$$D = D_1 + D_2. \quad (5)$$

Now, we will solving problems related to image formation and calculations in thin lenses.

Problem 1. Express the product x_1 and x_2 in terms of the focal length F of a thin lens, where x_1 is the distance from the first focal point to the object and x_2 is the distance from the second focal point to the real image.

Given:

x_1

x_2

Need to find

$F=?$

Solution. To solve the problem, we first draw the diagram and label the necessary quantities. After completing the drawing, we use formula (1) for finding the focal length of a thin lens, namely:

$$\frac{1}{F} = \frac{1}{a} + \frac{1}{b}.$$

In order to relate the focal length F in this formula to the given quantities, namely x_1 and x_2 , we write the following expressions (Fig. 1)

$$a = F + x_1, \quad b = F + x_2. \quad (6)$$

By substituting the expressions (6) for the quantities a and b in expression (1), we obtain the following expression

$$\frac{1}{F} = \frac{1}{F + x_1} + \frac{1}{F + x_2}. \quad (7)$$

By finding a common denominator for the right side of the resulting expression (7), we obtain the following

$$\frac{1}{F} = \frac{F + x_2 + F + x_1}{(F + x_1)(F + x_2)}. \quad (8)$$

By cross-multiplying the denominators of expression (8), we eliminate the fractions to obtain the following

$$(F + x_1)(F + x_2) = F(2F + x_1 + x_2).$$

By expanding the brackets in the above expression, we obtain the following

$$F^2 + Fx_2 + Fx_1 + x_1x_2 = 2F^2 + Fx_1 + Fx_2.$$

By simplifying and canceling out the like terms, we obtain the following expression

$$x_1x_2 = F^2. \quad (9)$$

Equation (9) is the required relation to be found.

Answer: $x_1x_2 = F^2$.

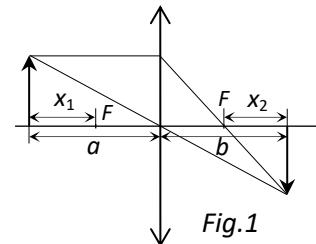


Fig.1

Problem 2. A lens with an optical power of 6D is made of glass with a refractive index of 1,6. When the lens is immersed in a liquid, it becomes a diverging lens with a focal length of 2 m. Determine the refractive index of the liquid.

Given:

$$D_1=6 \text{ D}$$

$$n_L=1,6$$

$$F_2=2 \text{ m}$$

Need to find

$$n_{lq}=?$$

Solution. To solve this problem, we apply formula (3) for optical power of a thin lens, namely

$$D = (n_L - n_m) \left(\frac{1}{R_1} + \frac{1}{R_2} \right).$$

We write the above expression (3) for the case where the lens is situated in air, namely

$$D_1 = (n_L - n_a) \left(\frac{1}{R_1} + \frac{1}{R_2} \right), \quad (10)$$

where, n_a – refractive index of air.

When the thin lens is immersed in an unknown liquid (Fig.2), it becomes a diverging lens. By applying expression (3) to this lens, we obtain the following

$$-D_2 = \frac{n_{lq}}{F_2} = (n_L - n_{lq}) \left(\frac{1}{R_1} + \frac{1}{R_2} \right), \quad (11)$$

where, n_{lq} – refractive index of liquid.

By dividing expression (11) by expression (10) term-by-term, we obtain

$$-\frac{n_{lq}}{F_2 D_1} = \frac{n_L - n_{lq}}{n_L - n_a}. \quad (12)$$

We rearrange expression (12) into the following form

$$-\frac{n_L - n_a}{F_2 D_1} = \frac{n_L - n_{lq}}{n_{lq}} = \frac{n_L}{n_{lq}} - 1. \quad (13)$$

From expression (13), we get the following

$$\frac{n_L}{n_{lq}} = 1 - \frac{n_L - n_a}{F_2 D_1}. \quad (14)$$

By solving expression (14) for the refractive index of the liquid, we obtain the following expression

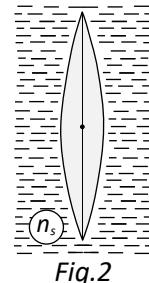


Fig.2

$$n_{lq} = \frac{n_L}{1 - \frac{n_L - n_a}{F_2 D_1}}. \quad (15)$$

Calculation:

$$n_s = \frac{1,6}{1 - \frac{1,6 - 1}{2 \cdot 6}} = 1,68.$$

Answer: $n_s = 1,68$.

Conclusion

To determine the focal length and optical power of lenses using ray tracing and the thin lens formula, it is essential to deeply understand the essence of these equations and their application to specific scenarios. Many students, and even school teachers, often find applying these formulas challenging, as such examples are frequently not well-explained online. For this reason, this article details how a converging lens can be transformed into a diverging lens by immersing it in a medium with a higher optical density than the lens material itself. We have provided a step-by-step solution to determine the refractive index of such a liquid. These examples will be beneficial for school students preparing for science olympiads, university students studying the optics branch of physics, as well as instructors and students at educational centers.

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