TO‘RTINCHI TARTIBLI MURAKKAB TURDAGI TENGLAMA UCHUN TO‘G‘RI TO‘RTBURCHAK SOHADA CHEGARAVIY MASALALAR

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Annotatsiya

Maqolada to‘rtinchi tartibli murakkab turdagi tenglamalar uchun chegaraviy masala qaralgan bo‘lib, masalaning yechimining yagonaligi integral energiya usulida, yechimning mavjudligini isbotlashda integral tenglamalari nazariyasidan foydalanilgan.

Kalit so‘zlar

Grin formulasi, yuqori tartibli tenglama, Laplas operatori, yechimning yagonaligi, yechimning mavjudligi, energiya integrali.

Annotation

The article considers a boundary value problem for a fourth-order equation of a complex type. The uniqueness of the solution is proven using the energy integral method, while the existence of the solution is established by applying the theory of integral equations.

Keywords

Green's formula, higher-order equation, Laplace operator, uniqueness of the solution, existence of the solution, energy integral.

Ma’lumki, murakkab turdagi tenglamalarni o‘rganish va ularni tahlil qilish usullarini yaratishda juda ko‘p olimlar shu jumaladan M.S.Salohiddinov[1,2], T.D.Jo‘rayev[3] va ularning shogirdlari katta hissa qo‘shishgan. Mazkur maqola ham yuqori tartibli tenglamalarni o‘rganishga bag‘ishlangan.

Ushbu

$$\frac{\partial^2}{\partial y^2}\Delta u + cu = f(x,y) \qquad (1)$$

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tenglamani $D=\{(x,y):0<x<1,0<y<1\}$ to'g'ri to'rtburchak sohada qaraymiz. Bu sohaning uchlari $A(0;0)$, $B(1;0)$, $C(1;1)$, $D(0;1)$ belgilaymiz.

Bunda $\Delta\equiv\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ - Laplas operatori, $c(x,y)$, $f(x,y)$ - berilgan funksiyalar.

Masalaning qo'yilishi. Shunday $u(x,y)\in C^2(\overline{D})\cap C^4(D)$ funksiyani topingki, u D sohada (1) tenglamani va quyidagi (2)-(4) shartlarni qanoatlantirsin:

$$u(0,y)=\varphi_0(y) \qquad u(1,y)=\varphi_1(y), \qquad 0<y<1$$
$$u(x,0)=\psi_0(x) \qquad u(x,1)=\psi_1(x), \qquad 0<x<1 \qquad (2)$$

$$u(x,-x+1)=\psi_2(x), \qquad 0<x<1 \qquad (3)$$

$$u_y(x,0)=\psi_3(x), \qquad 0<x<1 \qquad (4)$$

Bu yerda φ_i, ψ_i $(i=\overline{0,3})$ lar berilgan funksiyalar bo'lib, yechimni ko'rsatilgan sinfdan bo'lishini ta'minlovchi kelishuv shartlarini qanoatlantiradi.

Masala yechimining yagonaligi.

Teorema. Agar $c(x,y)\geq 0,(x,y)\in D$ va $\varphi_1'(1)=\varphi_0'(1)=0$ shartlar bajarilsa, u holda masalaning bittadan ortiq yechimi mavjud emas.

Isbot. Masala yechimining yagonaligini isbotlash uchun (1) - (4) masalaning yechimi 2 ta deb faraz qilamiz. u_1 va u_2 funksiyalar (1)-(4) masalaning yechimi bo'lsin. U holda

$$u=u_1-u_2$$

ham (1) tenglamani qanoatlantiradi. Bu yechimga mos chegaraviy shartlar bir jinsli bo'lib qoladi. Ya'ni

$$\varphi_i\equiv\psi_i\equiv f(x,y)\equiv 0, \qquad i=0,1,2,3 \qquad (5)$$

Ushbu D_ε soha $D_\varepsilon=\{(x,y):\varepsilon<x<1-\varepsilon,\varepsilon<y<1-\varepsilon\}$ ko'rinishida bo'lsin. Γ_ε - D_ε sohaning chegarasi.

Berilgan (1) tenglamani $u(x,y)$ ga ko'paytirib, D_ε soha bo'yicha integrallaymiz:

$$\iint_{D_\varepsilon} u(\frac{\partial^2}{\partial y^2}\Delta u+cu)dxdy=0 \qquad (6)$$

D_ε sohada quyidagi tenglik o'rinli:

$$u\frac{\partial^2}{\partial y^2}\Delta u=u\frac{\partial}{\partial y}\Delta u_y=\frac{\partial}{\partial y}(u\Delta u_y)-\left[\frac{\partial}{\partial x}(u_y u_{xy})-u_{xy}^2+\frac{\partial}{\partial y}(u_y u_{yy})-u_{yy}^2\right] \qquad (7)$$

(7) tenglikni (6) ga qo'yamiz va Grin formulasini qo'llaymiz:

$$\int_{\Gamma_\varepsilon} u\Delta u_y dx - \int_{\Gamma_\varepsilon} u_y u_{xy} dy - \int_{\Gamma_\varepsilon} u_y u_{yy} dx + \iint_{D_\varepsilon} (u_{xy}^2 + u_{yy}^2 + c(x,y)u^2) dxdy = 0 \quad (8)$$

Bu yerda (5) ni inobatga olib, (2)-(4) chegaraviy shartlardan foydalanamiz va har bir integralni alohida hisoblaymiz:

$$J_1 = \int_{\Gamma_\varepsilon} u\Delta u_y dx$$

$\varepsilon \to 0$ da bir jinsli shartlarga asosan $J_1 = 0$ tenglik o‘rinli.

(8) tenglikning 2-hadidagi integralda AB va CD da $dy = 0$ bo‘ladi. U holda quyidagi

$$J_2 = \int_{\Gamma_\varepsilon} u_y u_{xy} dy = \int_{BC} u_y u_{xy}(1,y) dy + \int_{DA} u_y u_{xy}(0,y) dy$$

tenglik o‘rinli. Bir jinsli shartga asosan $u_y = 0$ bo‘lib, $J_2 = \int_{\Gamma_\varepsilon} u_y u_{xy} dy = 0$ ekanligi kelib chiqadi.

BC va DA chiziqlarda $u_y u_{yy}$ dan olingan integralni qarasak, BC va DA da $dx = 0$. Bundan

$$J_3 = \int_{\Gamma_\varepsilon} u_y u_{yy} dx = \int_{AB} u_y u_{yy}(x,0) dx + \int_{CD} u_y u_{yy}(x,1) dx$$

tenglik o‘rinli. Chegaraviy shartlarni hisobga olib,

$$\int_{AB} u_y u_{yy}(x,0) dx + \int_{CD} u_y u_{yy}(x,1) dx = \int_{CD} u_y u_{yy}(x,1) dx$$

ko‘rinishga kelamiz. Bu integralda (3) shartdan foydalanamiz, ya’ni $x = -y + 1$ va $dy = -dx$ almashtirish bajaramiz, hamda teorema shartiga asosan $\varphi_1'(1) = \varphi_0'(1) = 0$ ekanligidan

$$\int_0^1 u_y(x,1) u_{yy}(x,1) dx = 0$$

tenglikka kelamiz. U holda (8) ifoda $\varepsilon \to 0$ da

$$\iint_D (u_{xy}^2 + u_{yy}^2) dxdy + \iint_D c(x,y) u^2 dxdy = 0 \quad (9)$$

ko‘rinishni oladi.

Oxirgi (9) tenglikda:

a) $c(x,y) > 0$ bo‘lsa, u holda $u \equiv 0$ ekani kelib chiqadi;

b) $c(x,y) = 0$ bo‘lsa, D sohada $u_{xy} = u_{yy} = const$ bo‘ladi. $u \in C^2(\bar{D})$ ekanligidan, $u_{xy} = u_{yy} = 0$ tenglikni olish mumkin.

$u_{yy}=0$ tenglamani integrallab, $u=f(x)y+f_1(x)$ yechimga ega bo'lamiz. Bir jinsli chegaraviy shartlarga asosan $u\equiv 0$ ekani kelib chiqadi. Demak $u_1=u_2$ bo'lib, masala yechimi yagona bo'ladi. **Teorema isbotlandi.**

Masala yechimining mavjudligi.

Teorema. Agar quyidagi

$\varphi_i''(y),(0\le y\le 1),\psi_i''(x),i=0,1,\quad \psi_3(x),\psi_2(x),\psi_3'(x),\psi_2'(x),\ f(x,y)$

$0\le x\le 1,\quad 0\le y\le 1$ oraliqlarda funksiyalar uzluksiz bo'lib,

$\varphi_1'(1)=\varphi_0'(1)=0$

bo'lsa, masala yechimi mavjud.

Isbot. $u_{yy}=\upsilon$ belgilash yordamida (1) tenglama quyidagi ko'rinishni oladi:

$$\Delta\upsilon=f(x,y)-c(x,y)u(x,y) \qquad (10)$$

(10) tenglama uchun masalaning (2), (3) va (4) chegaraviy shartlaridan foydalanib quyidagicha shartlarga ega bo'lamiz:

$$\begin{aligned}
&\upsilon(0,y)=u_{yy}(0,y)=\varphi_0''(y)\\
&\upsilon(x,0)=u_{yy}(x,0)=\overline{\psi_3}(x)\\
&\upsilon(1,y)=u_{yy}(1,y)=\varphi_1''(y)\\
&\upsilon(x,1)=u_{yy}(x,1)=\overline{\psi_1}(x)
\end{aligned} \qquad (11)$$

Bu yerda $u_{yy}(x,1)=\overline{\psi_1}(x)$ va $u_{yy}(x,0)=\overline{\psi_3}(x)$ - funksiyalar noma'lum.

(10) tenglamaning (11) chegaraviy shartlarni qanoatlantiruvchi yechimi quyidagicha bo'ladi[5]:

$$\begin{aligned}
\upsilon(x,y)=&\int_0^1 G_\eta(x,y,\xi,0)\overline{\psi_3}(\xi)d\xi-\int_0^1 G_\eta(x,y,\xi,1)\overline{\psi_1}(\xi)d\xi-\\
&-\int_0^1 G_\xi(x,y,1,\eta)\varphi_1''(\eta)d\eta+\int_0^1 G_\xi(x,y,0,\eta)\varphi_0''(\eta)d\eta-\\
&-\iint_D G(x,y,\xi,\eta)c(\xi,\eta)u(\xi,\eta)d\xi d\eta+F(x,y)
\end{aligned} \qquad (12)$$

Bu yerda

$$F(x,y)=\iint_D G(x,y,\xi,\eta)f(\xi,\eta)d\xi d\eta,$$

$$G(x,y,\xi,\eta)=-\ln r+g(x,y,\xi,\eta),\quad \left(r=\sqrt{(x-\xi)^2+(y-\eta)^2}\right)$$

Laplas tenglamasi uchun to'rtburchak sohada Dirixle masalasining Grin funksiyasi.

Yuqoridagi belgilashga asosan quyidagi 2 ta masalaga ega boʻlamiz:

$$\begin{cases} w_y = \upsilon(x,y) \\ w(x,0) = \psi_3(x) \end{cases} \tag{13}$$

va

$$\begin{cases} u_y = \upsilon(x,y) \\ u(x,-x+1) = \psi_2(x) \end{cases} \tag{14}$$

U holda (13) masalaning yechimi:

$$w = \int_0^y \upsilon(x,t)dt + \psi_3(x) \tag{15}$$

koʻrinishida boʻladi. Bu ifodadan foydalanib, (14) ni yechimini quyidagicha yozamiz:

$$u = \int_{1-x}^{y} w(x,t)dt + \psi_2(x) \tag{16}$$

Endi (15) ga (12) tenglikni qoʻysak:

$$w = \int_0^y \left[\int_0^1 G_\eta(x,t,\xi,0)\overline{\psi_3}(\xi)d\xi - \int_0^1 G_\eta(x,t,\xi,1)\overline{\psi_1}(\xi)d\xi - \right.$$

$$\left. -\int_0^1 G_\xi(x,t,1,\eta)\varphi_1''(\eta)d\eta + \int_0^1 G_\xi(x,t,0,\eta)\varphi_0''(\eta)d\eta \right]dt -$$

$$-\int_0^y \iint_D G(x,t,\xi,\eta)c(\xi,\eta)u(\xi,\eta)d\xi d\eta dt + \psi_3(x) +$$

$$+\int_0^y \left[\iint_D G(x,t,\xi,\eta) f(\xi,\eta)d\xi d\eta \right]dt \tag{17}$$

koʻrinishga keladi.

(17) ifodani (16) formulaga qoʻyib, quyidagiga ega boʻlamiz:

$$u = \int_{1-x}^{y} \left[\int_0^t \left[\int_0^1 G_\eta(x,t_1,\xi,0)\overline{\psi_3}(\xi)d\xi - \int_0^1 G_\eta(x,t_1,\xi,1)\overline{\psi_1}(\xi)d\xi - \right.\right.$$

$$\left.\left. -\int_0^1 G_\xi(x,t_1,1,\eta)\varphi_1''(\eta)d\eta + \int_0^1 G_\xi(x,t_1,0,\eta)\varphi_0''(\eta)d\eta \right]dt_1 \right]dt - \tag{18}$$

$$-\int_{1-x}^{y} \left[\int_0^t \iint_D G(x,t_1,\xi,\eta)c(\xi,\eta)u(\xi,\eta)d\xi d\eta dt_1 - \psi_3(x) \right]dt +$$

$$+\int_{1-x}^{y}\int_{0}^{t}\left[\iint_{D} G(x,t_1,\xi,\eta)\, f(\xi,\eta)\,d\xi d\eta\right]dt_1 dt+\psi_2(x)$$

(18) tenglikdagi ba’zi integrallarni hisoblaymiz:

$$J_0=\int_{1-x}^{y}\left(\int_{0}^{t} G(x,t_1,\xi,\eta)\,dt_1\right)dt=\int_{1-x}^{y}\int_{0}^{t}\left(-\ln\sqrt{(x-\xi)^2+(t_1-\eta)^2}+g(x,t_1,\xi,\eta)\right)dt_1 dt=$$

$$=-\frac{1}{4}\left((x-\xi)^2+(y-\eta)^2\right)\ln\left((x-\xi)^2+(y-\eta)^2\right)+g_1(x,y,\xi,\eta)$$

$$J_1=\int_{1-x}^{y}\int_{0}^{1}\left[\int_{0}^{t} G_\eta(x,t_1,\xi,1)\overline{\psi}_1(\xi)\,d\xi\right]dt_1 dt=\int_{1-x}^{y}\left[\int_{0}^{1}\overline{\psi}_1(\xi)\,d\xi\int_{0}^{t} G_\eta(x,t_1,\xi,1)dt_1\right]dt=$$

$$=\int_{0}^{1}\overline{\psi}_1(\xi)\left[(y-1)\ln\left[\frac{(x-\xi)^2+(y-1)^2}{(x-\xi)^2+1}\right]+x\ln\left[\frac{(x-\xi)^2+x^2}{(x-\xi)^2+1}\right]+g_2(x,y,\xi,1)\right]d\xi$$

$$J_2=\int_{1-x}^{y}\int_{0}^{1}\left[\int_{0}^{t} G_\eta(x,t_1,\xi,0)\overline{\psi}_3(\xi)\,d\xi\right]dt_1 dt=\int_{1-x}^{y}\left[\int_{0}^{1}\overline{\psi}_3(\xi)\,d\xi\int_{0}^{t} G_\eta(x,t_1,\xi,0)dt_1\right]dt=$$

$$=\int_{0}^{1}\overline{\psi}_3(\xi)\left[y\ln\left[\frac{(x-\xi)^2+y^2}{(x-\xi)^2}\right]+(1-x)\ln\left[\frac{(x-\xi)^2+(1-x)^2}{(x-\xi)^2}\right]+g_3(x,y,\xi,0)\right]d\xi$$

bu yerda

$$g_1=-\frac{1}{2}\int_{1-x}^{y}\left[\eta\ln\left((x-\xi)^2+\eta^2\right)-2(t-\eta)-2\eta+2arctg\frac{t-\eta}{x-\xi}-2arctg\frac{-\eta}{x-\xi}\right]dt+$$

$$+\int_{1-x}^{y}\int_{0}^{t} g(x,t_1,\xi,\eta)dt_1 dt$$

$$g_2=-2(y-x-1)+2(x-\xi)\left(arctg\frac{y-1}{x-\xi}-arctg\frac{-x}{x-\xi}\right)+\int_{1-x}^{y}\int_{0}^{t} g_\eta(x,t_1,\xi,1)dt_1 dt$$

$$g_3=-2(y+x-1)+2(x-\xi)\left(arctg\frac{y}{x-\xi}-arctg\frac{1-x}{x-\xi}\right)+\int_{1-x}^{y}\int_{0}^{t} g_\eta(x,t_1,\xi,0)dt_1 dt$$

Topilganlardan foydalanib, $u(x,y)$ ni soddalashtiramiz:

$$u(x,y)+\frac{1}{4}\iint_{D} R(x,y,\xi,\eta)u(\xi,\eta)\,d\xi d\eta=\overline{F}(x,y) \qquad (19)$$

Bu yerda

$$\overline{F}(x,y)=F_1(x,y)+F_2(x,y)$$

$$F_1(x,y)=\frac{1}{4}\iint_D R(x,y,\xi,\eta)\,f(\xi,\eta)\,d\xi d\eta$$

$$F_2(x,y)=\int_0^1\left[\int_{1-x}^{y}\int_0^t G_\xi(x,t_1,0,\eta)dt_1dt\right]\varphi_0''(\eta)\,d\eta-\int_0^1\left[\int_{1-x}^{y}\int_0^t G_\xi(x,t_1,1,\eta)dt_1dt\right]\varphi_1''(\eta)\,d\eta-$$

$$-\int_0^1\overline{\psi_1}(\xi)\left[(y-1)\ln\left[\frac{(x-\xi)^2+(y-1)^2}{(x-\xi)^2+1}\right]+x\ln\left[\frac{(x-\xi)^2+x^2}{(x-\xi)^2+1}\right]+g_2(x,y,\xi,1)\right]d\xi+$$

$$+\int_0^1\overline{\psi_3}(\xi)\left[y\ln\left[\frac{(x-\xi)^2+y^2}{(x-\xi)^2}\right]+(1-x)\ln\left[\frac{(x-\xi)^2+(1-x)^2}{(x-\xi)^2}\right]+g_3(x,y,\xi,0)\right]d\xi+$$

$$+(y-1+x)\psi_3(x);$$

$$R(x,y,\xi,\eta)=\left[\left((x-\xi)^2+(y-\eta)^2\right)\ln\left((x-\xi)^2+(y-\eta)^2\right)+g_1(x,y,\xi,\eta)\right]c(\xi,\eta)$$

(19) – Fredgolmning 2-tur integral tenglamasi bo‘lib, ko‘rish mumkinki, uning yadrosi $R(x,y,\xi,\eta)$ - uzluksiz funksiya, $\overline{F}(x,y)$ funksiya ham chegaraviy shartlardagi berilgan funksiyalarga qo‘yilgan shartlarga asosan uzluksiz bo‘ladi. Shuning uchun, unga Fredgolm teoremalarini qo‘llash mumkin. Shunday qilib, uning yechimi[1]:

$$u(x,y)=\overline{F}(x,y)-\frac{1}{4}\iint_D \Gamma(x,y,\xi,\eta)\overline{F}(\xi,\eta)\,d\xi d\eta \qquad (20)$$

ko‘rinishida bo‘ladi. Bu yerda $\Gamma(x,y,\xi,\eta)-R(x,y,\xi,\eta)$ yadroning rezolventasi.

$\overline{F}(x,y)$ ni ifodasidan foydalanib, (20) ni quyidagicha yozamiz:

$$u(x,y)=-\int_0^1\overline{\psi_1}(\xi)\left[(y-1)\ln\left[\frac{(x-\xi)^2+(y-1)^2}{(x-\xi)^2+1}\right]+x\ln\left[\frac{(x-\xi)^2+x^2}{(x-\xi)^2+1}\right]+g_2(x,y,\xi,1)\right]d\xi+$$

$$+\int_0^1\overline{\psi_3}(\xi)\left[y\ln\left[\frac{(x-\xi)^2+y^2}{(x-\xi)^2}\right]+(1-x)\ln\left[\frac{(x-\xi)^2+(1-x)^2}{(x-\xi)^2}\right]+g_3(x,y,\xi,0)\right]d\xi-$$

$$-\iint_D \Gamma(x,y,\xi,\eta)\left[\int_0^1 \overline{\psi_1}(\xi_1)\left[(\eta-1)\ln\left[\frac{(\xi-\xi_1)^2+(\eta-1)^2}{(x-\xi_1)^2+1}\right]+\xi\ln\left[\frac{(\xi-\xi_1)^2+\xi^2}{(\xi-\xi_1)^2+1}\right]+g_2(\xi,\eta,\xi_1,1)\right]\right]d\xi_1 d\xi d\eta+$$

$$+\iint_D \Gamma(x,y,\xi,\eta)\left[\int_0^1 \overline{\psi_3}(\xi_1)\left[\eta\ln\left[\frac{(\xi-\xi_1)^2+\eta^2}{(\xi-\xi_1)^2}\right]+(1-\xi)\ln\left[\frac{(\xi-\xi_1)^2+(1-\xi)^2}{(\xi-\xi_1)^2}\right]+g_3(\xi,\eta,\xi_1,0)\right]\right]d\xi_1 d\xi d\eta+$$

$$+F_3(x,y) \qquad (21)$$

Bu yerda

$$F_3(x,y)=\frac{1}{4}\iint_D R(x,y,\xi,\eta)f(\xi,\eta)d\xi d\eta+\int_0^1\left[\int_{1-x}^{y}\int_0^t G_\xi(x,t_1,0,\eta)dt_1dt\right]\varphi_0''(\eta)d\eta-$$

$$-\int_0^1\left[\int_{1-x}^{y}\int_0^t G_\xi(x,t_1,1,\eta)dt_1dt\right]\varphi_1''(\eta)d\eta+(y-1+x)\psi_3(x)$$

$u_{yy}(x,1)=\overline{\psi_1}(x)$ ekanligidan foydalanib, (21) da y bo‘yicha ikki marta hosila olamiz va $y=1$ qo‘ysak,

$$\overline{\psi_1}(x)=\int_0^1 R_1(x,\xi_1)\overline{\psi_1}(\xi_1)d\xi_1+\int_0^1 k_1(x,\xi_1)\overline{\psi_3}(\xi_1)d\xi_1+F_3(x) \qquad (22)$$

ko‘rinishidagi tenglamaga kelamiz. Bu yerda

$$R_1(x,\xi)=\iint_D \Gamma_{yy}(x,1,\xi_1,1)\left[(1-\eta)\ln\left[\frac{(\xi-\xi_1)^2+(\eta-1)^2}{(x-\xi_1)^2+1}\right]-\xi\ln\left[\frac{(\xi-\xi_1)^2+\xi^2}{(\xi-\xi_1)^2+1}\right]-g_2(\xi,\eta,\xi_1,1)\right]d\xi d\eta$$

$$k_1(x,\xi_1)=\frac{2}{(x-\xi_1)^2+1}+\frac{4(x-\xi_1)^2}{\left((x-\xi_1)^2+1\right)^2}+\iint_D \Gamma_{yy}(x,1,\xi,\eta)\left[\eta\ln\left[\frac{(\xi-\xi_1)^2+\eta^2}{(\xi-\xi_1)^2}\right]+\right.$$

$$\left.+(1-\xi)\ln\left[\frac{(\xi-\xi_1)^2+(1-\xi)^2}{(\xi-\xi_1)^2}\right]+g_3(\xi,\eta,\xi_1,0)\right]d\xi d\eta$$

$$F_3(x)=F_{3yy}(x,1)$$

Endi $u_{yy}(x,0)=\overline{\psi_3}(x)$ ekanligidan (21) dan y bo‘yicha ikki marta hosila olib, $y=0$ ni o‘rniga qo‘yib,

$$\overline{\psi_3}(x)=\int_0^1 R_2(x,\xi_1)\overline{\psi_3}(\xi_1)d\xi_1+\int_0^1 k_2(x,\xi_1)\overline{\psi_1}(\xi_1)d\xi_1+\overline{F_3}(x) \qquad (23)$$

tenglamaga kelamiz. Bu yerda

$$R_2(x,\xi)=\iint_D \Gamma_{yy}(x,0,\xi,\eta)\left[\eta\ln\left[\frac{(\xi-\xi_1)^2+\eta^2}{(\xi-\xi_1)^2}\right]+(1-\xi)\ln\left[\frac{(\xi-\xi_1)^2+(1-\xi)^2}{(\xi-\xi_1)^2}\right]+g_3(\xi,\eta,\xi_1,0)\right]d\xi d\eta$$

$$k_2(x,\xi_1)=\frac{2}{(x-\xi_1)^2+1}+\frac{4(x-\xi_1)^2}{\left((x-\xi_1)^2+1\right)^2}-\iint_D \Gamma_{yy}(x,1,\xi,\eta)\left[(\eta-1)\ln\left[\frac{(\xi-\xi_1)^2+(\eta-1)^2}{(x-\xi_1)^2+1}\right]+\right.$$

$$\left.+\xi\ln\left[\frac{(\xi-\xi_1)^2+\xi^2}{(\xi-\xi_1)^2+1}\right]+g_2(\xi,\eta,\xi_1,1)\right]d\xi d\eta$$

$$\overline{F_3}(x)=F_{3yy}(x,0)$$

Shunday qilib, nomaʻlum funksiyalarga nisbatan Fredgolmning 2-tur integral tenglamalarini hosil qilamiz. Berilgan funksiyalarga qoʻyilgan shartlarga asosan (22), (23) tenglamalarni oʻng tomoni uzluksiz funksiyalar boʻlib, $x=0$, $x=1$ nuqtalarda chegaralangan boʻladi.

(22) va (23) integral tenglamalardan $\overline{\psi_1}(x)$, $\overline{\psi_3}(x)$ nomaʻlumlarni topib, (21) ifodaga olib borib, berilgan masala yechimini topamiz.

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